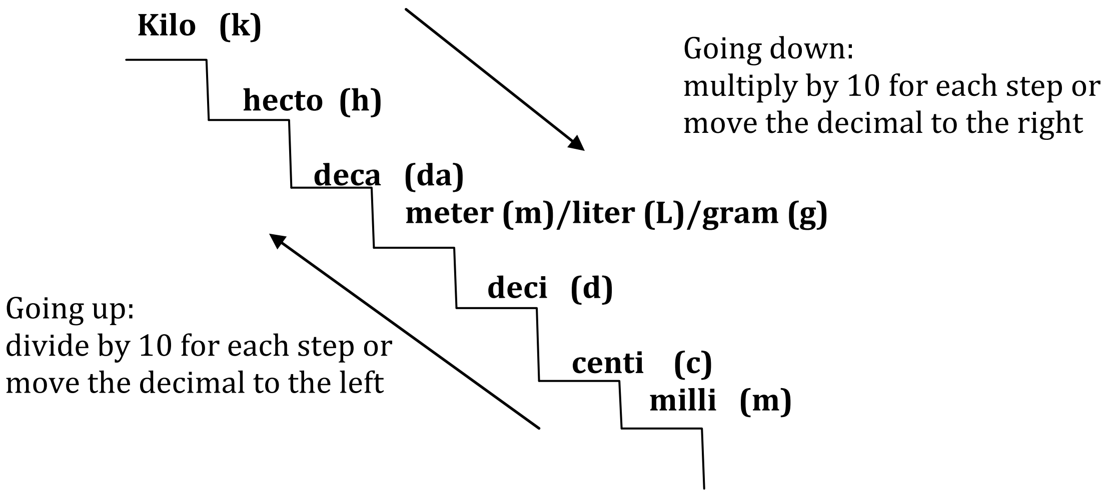
Unit 7: Similarity and Transformations **Grade 9 Math**

Whenever setting up numbers in a ratio or fraction, both numbers MUST have the same units. Know how to change units.



Examples

1. What is 300 m in centimeters?

    300 m = 30 000 cm

2. Write 1 cm represents 5 m as a ratio.

    * change units: 5 m = 500 cm 1 : 500 as a ratio

3. If a map scale tells you that 1 cm represents 15 km. What is 15 km in centimeters? Write the answer as a ratio.

    15 km = 1 500 000 1: 1 500 000 as a ratio

4. What is 7.5m in centimeters?

    7.5 m = 750 cm

5. How many kilometers does 3 750 000 cm represent?

    3 750 000 cm = 37.5 km

Section 7.1 Scale Diagrams and Enlargements

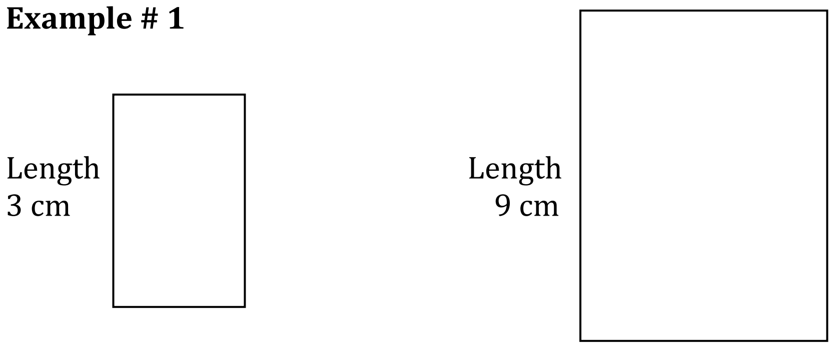
A diagram that is an **enlargement** or a **reduction** of another diagram is called a **scale diagram**.

The **scale factor** is the relationship between the matching lengths on the two diagrams.

To find the scale factor of a scale diagram,

we divide: $\dfrac{\text{length of the scale diagram}}{\text{length of the original object}}$

Example # 1



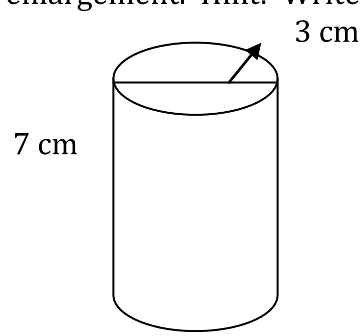
Original Scale Diagram

Scale factor = $\dfrac{\text{length on scale diagram}}{\text{length on original diagram}} = \dfrac{9}{3} = 3$

Note
- the **units must be the same** on the original and scale diagram
- if not, you must convert one to make them the same
- scale factors **do not have units.**

Example # 2

The cylinder is to be enlarged by a scale factor of $\frac{5}{2}$. Find the dimensions of the enlargement. Hint: Write the scale factor as a decimal.



Answer: Rewrite scale factor: $\frac{5}{2} = 5 \div 2 = 2.5$ Multiply each dimension by the scale factor.

Diameter Original: 3 cm   Diameter Enlargement: $3 \times 2.5 = 7.5$ cm

Height Original: 7 cm   Height Enlargement: $7 \times 2.5 = 17.5$ cm

The enlargement has diameter 7.5 cm and height 17.5cm.

Try this one!
A photo has dimensions 10cm by 15cm. Two enlargements are to be made with each scale factor below. Find the dimensions of each enlargement.

A) scale factor 4   B) scale factor $\frac{13}{4}$

Answer

A) Scale Factor = 4

Original Width: 10cm
Enlargement Width: 10cm × 4 = 40cm

Original Length: 15cm
Scale Length: 15cm × 4 = 60cm

Enlargement has dimensions 40cm by 60cm

B) Scale Factor = $\frac{13}{4} = 13 \div 4 = 3.25$

Original Width: 10cm
Enlargement Width: 10cm × 3.25 = 32.5cm

Original Length: 15cm
Enlargement Length: 15cm × 3.25 = 48.75cm

Enlargement has dimensions 32.5cm by 48.75cm

Enlargement examples so far:

scale ratio: 3

$\frac{5}{2} = 2.5$

4

$\frac{13}{4} = 3.25$

Notice the scale ratio for enlargements is always greater than 1

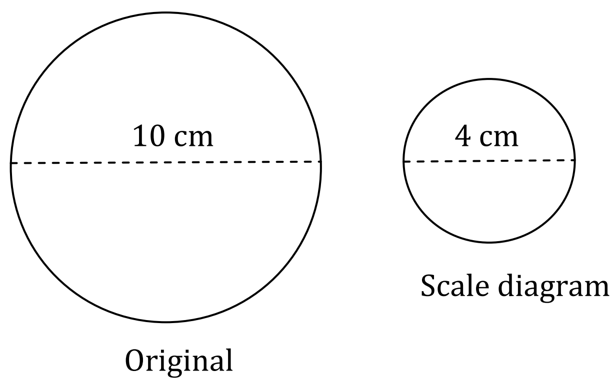
Section 7.2 Scale Diagrams and Reductions

A scale diagram can be smaller than the original diagram. This type of scale diagram is called a **reduction**.

A reduction has **a scale factor between 0 and 1.**

Example #1

a). What is the scale factor?
b). Is this an enlargement or a reduction?



Answer:
a). Scale factor = $\frac{\text{scale diagram}}{\text{original}}$

$= \frac{4}{10} = 0.4$

b). Reduction

Example # 2

A top view of a patio table is 105 cm by 165 cm. A reduction is to be drawn with scale factor of $\frac{1}{5}$. Find the dimensions of the reduction.

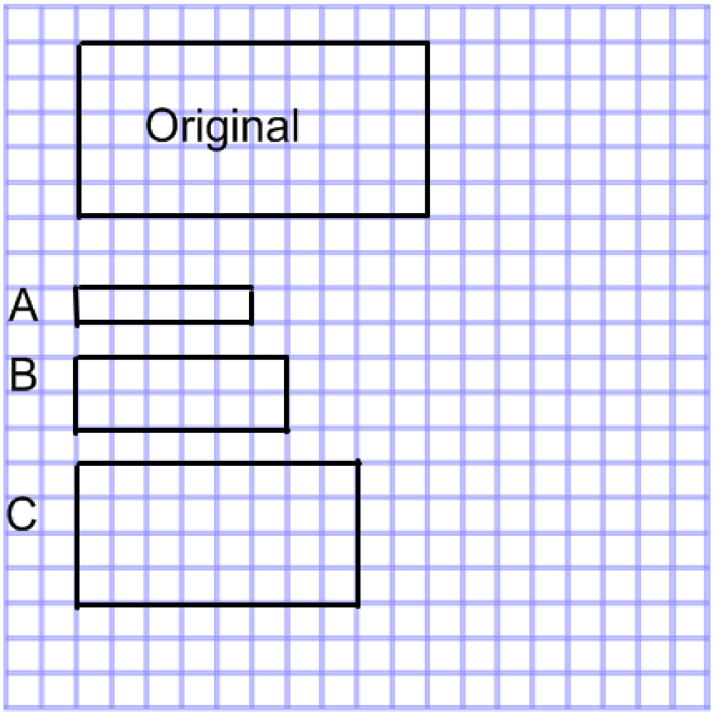
Answer:
Write the scale factor as a decimal $\frac{1}{5} = 0.2$

Original Width: 105 cm
Reduction Width: $105 \times 0.2 = 21$ cm

Original Length: 165 cm
Reduction Length: $165 \times 0.2 = 33$ cm

Dimensions of the reduction are 21cm by 33cm

Example #3: Which diagram has sides that are **proportional** to the original?



Proportion = means that 2 ratios are equal.

For example: an equation $\frac{3}{4} = \frac{6}{8}$ is a proportion.

Two diagrams are **proportional** if all sides are multiplied or divided by the same number.

Answer:

Original : 5 by 10

Write as a fraction and reduce: $\frac{5}{10} = \frac{1}{2}$

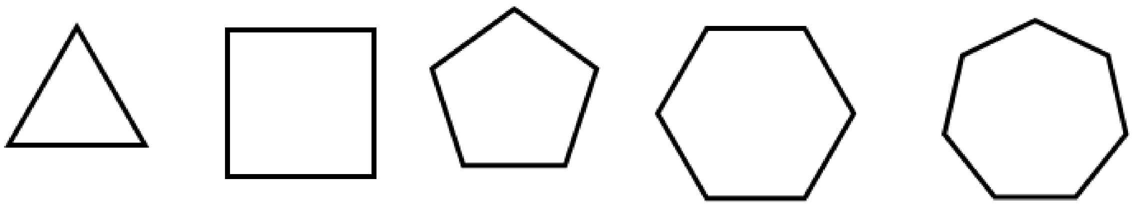
A). 1 by 5 $\frac{1}{5} \neq \frac{1}{2}$ not proportional

B). 2 by 6 $\frac{1}{3} \neq \frac{1}{2}$ not proportional

C). 4 by 8 $\frac{4}{8} = \frac{1}{2}$ is proportional

Section 7.3 **Similar Polygons**

Polygon is a closed shape with straight sides. Exactly 2 sides meet at a vertex.
Regular Polygon has equal sides and equal angles.

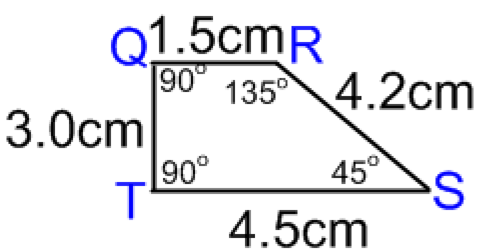


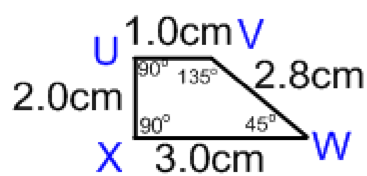
When one polygon is an enlargement or reduction of another polygon, we say the polygons are similar.

When 2 polygons are similar:

- Matching angles are equal AND
- Matching sides are proportional.

Example #1: Are these polygons similar?





Answer: Check matching angles: $\angle Q = \angle U = 90^0$ $\angle S = \angle W = 45^0$
$\angle R = \angle V = 135^0$ $\angle T = \angle X = 90^0$

Check matching sides:

$$\frac{QR}{UV} = \frac{1.5cm}{1.0cm} = 1.5$$

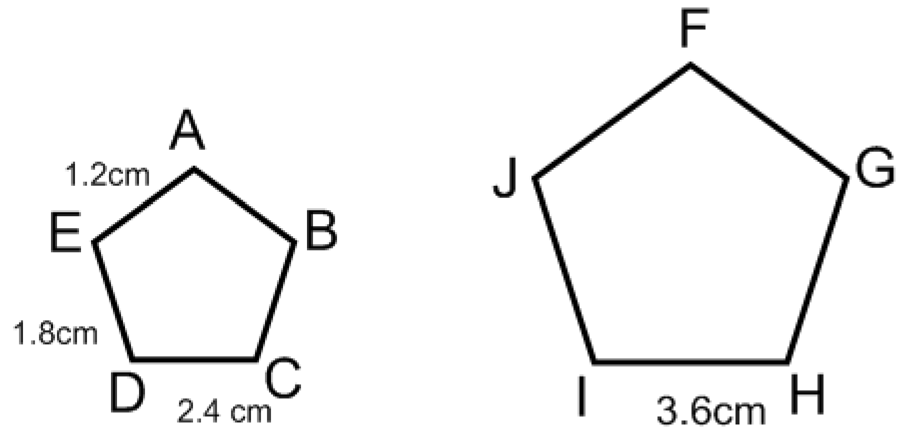
$$\frac{RS}{VW} = \frac{4.2cm}{2.8cm} = 1.5$$

$$\frac{ST}{WX} = \frac{4.5cm}{3.0cm} = 1.5$$

$$\frac{TQ}{XU} = \frac{3.0cm}{2.0cm} = 1.5$$

All scale factors are equal, so matching sides are proportional.
These figures are similar.

Example # 2: Use proportional method.
The following figures **are similar**. Determine the length of JI and JF



Answer:

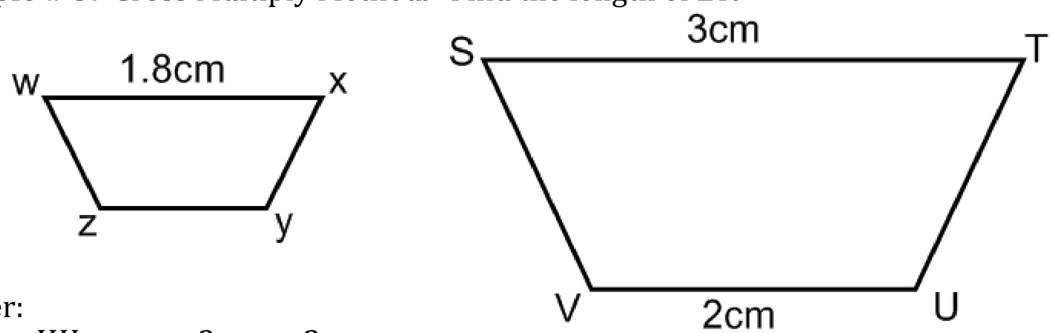
Determine the scale factor: $\frac{HI}{CD} = \frac{3.6}{2.4} = 1.5$

Multiply corresponding sides in the original by the scale factor.
JI corresponds with ED. JF corresponds with EA.

JI = 1.8 cm × 1.5 = 2.7 cm

JF = 1.2 cm × 1.5 = 1.8 cm

Example # 3: Cross Multiply Method. Find the length of ZY.



Answer:

$\frac{ST}{WX} = \frac{VU}{ZY} \longrightarrow \frac{3}{1.8} = \frac{2}{ZY}$

$\frac{3}{1.8} = \frac{2}{ZY}$

3(ZY) = 2 × 1.8

$\frac{3\ ZY}{3} = \frac{3.6}{3}$

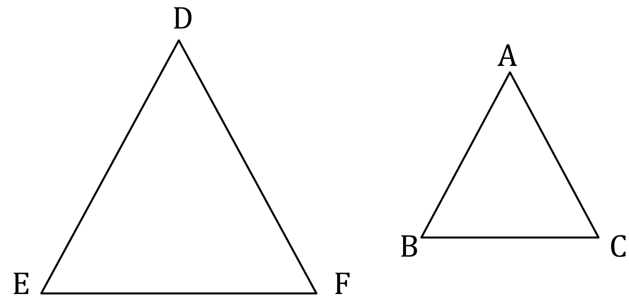
ZY = 1.2 cm

Section 7.4 **Similar Triangles** **Grade 9 Math**

Two triangles are similar if **they have the same shape but different size**.

In similar triangles:

- Matching angles are equal.
- Matching sides are proportional.



To write the similarity statement, corresponding angles and sides must match up.

$\Delta ABC \sim \Delta DEF$

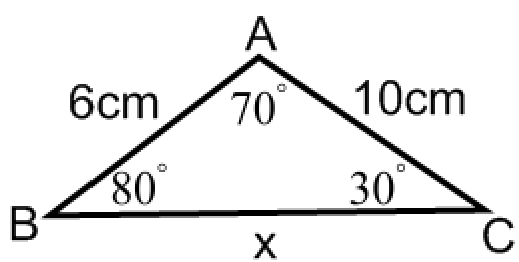
Can you write 6 true statements from the similarity of the two triangles?

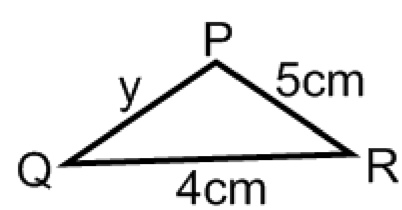
1. $\angle A = \angle D$
2. $\angle B = \angle E$
3. $\angle C = \angle F$
4. AB ∽ DE (is proportional to)
5. BC ∽ EF (is proportional to)
6. AC ∽ DF (is proportional to)

When writing proportions for corresponding sides, make sure to keep the **same triangle on top** in each fraction.

Example #1:

If $\Delta ABC \sim \Delta PQR$, find the angle of measures of ΔPQR and the missing side measurements of x and y.





Answer:

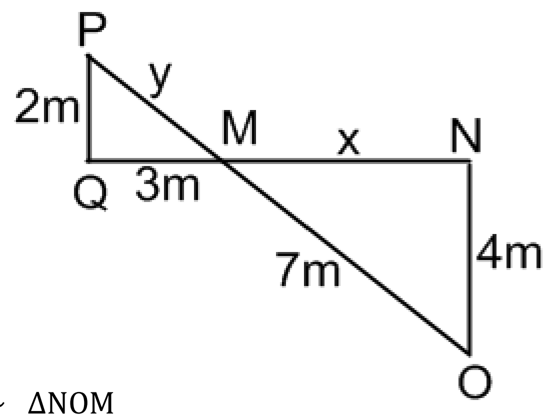
$\angle A = \angle P = 70^0$
$\angle B = \angle Q = 80^0$
$\angle C = \angle R = 30^0$

Write a proportion that includes only 1 unknown. Cross multiply and divide to solve.

$$\frac{AC}{PR}=\frac{BC}{QR}$$
$$\frac{10}{5}=\frac{x}{4}$$
$$(5)(x)=(10)(4)$$
$$5x=40$$
$$x=\frac{40}{5}$$
$$x=8cm$$

$$\frac{AC}{PR}=\frac{AB}{PQ}$$
$$\frac{10}{5}=\frac{6}{y}$$
$$(10)(y)=(6)(5)$$
$$10y=30$$
$$y=\frac{30}{10}$$

Example #2 Identify the 2 similar triangles and determine the missing sides.



Answer:

Match corresponding angles:
∠0 = ∠P
∠M = ∠M
∠N = ∠Q

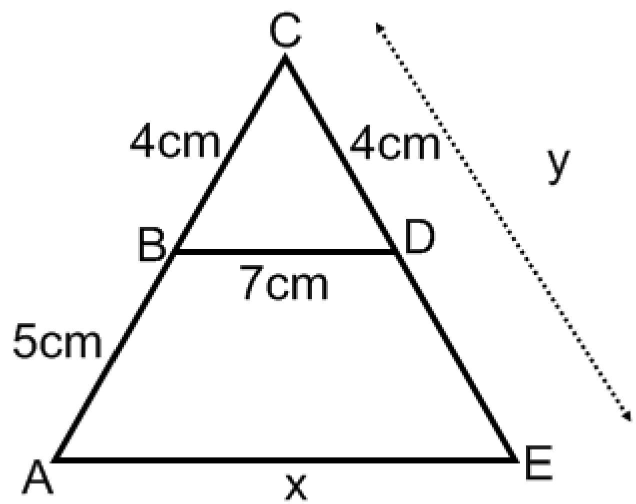
Write the similarity statement: ΔQPM ~ ΔNOM

Write a proportion that includes only 1 unknown. Cross multiply and divide to solve.

$$\frac{PQ}{ON}=\frac{QM}{NM}$$
$$\frac{2}{4}=\frac{3}{x}$$
$$(2)(x)=(3)(4)$$
$$2x=12$$
$$x=\frac{12}{2}$$
$$x=6m$$

$$\frac{PQ}{ON}=\frac{PM}{OM}$$
$$\frac{2}{4}=\frac{y}{7}$$
$$(4)(y)=(2)(7)$$
$$4y=14$$
$$y=\frac{14}{4}$$
$$y=3.5m$$

Example # 3 Identify the similar triangles and identify the missing measures.



Answer:
Match corresponding angles and write the similarity statement.

$\angle A = \angle B$
$\angle C = \angle C$
$\angle E = \angle D$

$\Delta ACE \sim \Delta BCD$

Find the length of side y:

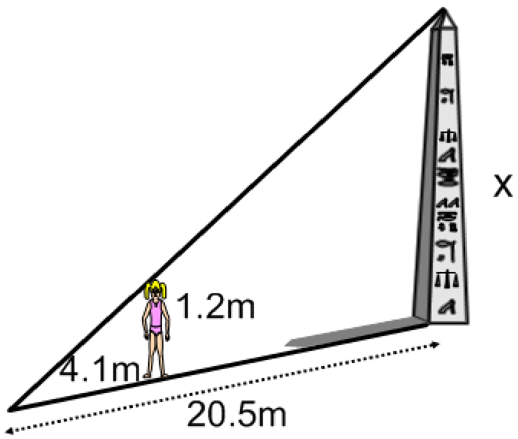
$$\frac{BC}{AC} = \frac{DC}{EC}$$
$$\frac{4}{9} = \frac{4}{y}$$
$$\therefore y = 9cm$$

Find the length of side x:

$$\frac{BC}{AC} = \frac{BD}{AE}$$
$$\frac{4}{9} = \frac{7}{x}$$
$$(4)(x) = (7)(9)$$
$$4x = 63$$
$$x = \frac{63}{4}$$
$$x = 15.75cm$$

Similar Triangles and Word Problems

#1: The length of a monument's shadow is 20.5m, when the length of Joan's shadow is 4.1m. If Joan is 1.2m tall, calculate the height of the monument.



Answer:

$$\frac{\text{Joan's Shadow}}{\text{Monument's Shadow}} = \frac{\text{Joan's Height}}{\text{Monument's Height}}$$

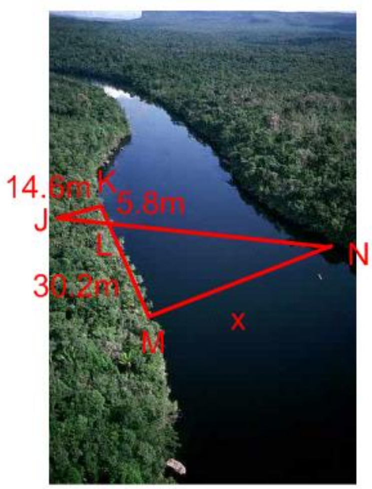
$$\frac{4.1}{20.5} = \frac{1.2}{x}$$

4.1x = (20.5)(1.2)

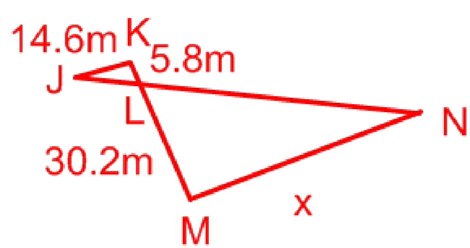
$$\frac{4.1x}{4.1} = \frac{24.6}{4.1}$$

x = 6 m is the height of the monument

#2: To measure the width of a river the measurements shown were made by a surveyor.



How will she determine the width of the river?



When working with decimals, round to the nearest tenth. (One # after the decimal)

Answer:

$$\frac{KL}{ML} = \frac{JK}{NM} \qquad \frac{5.8}{30.2} = \frac{14.6}{x}$$

5.8x = (14.6)(30.2)

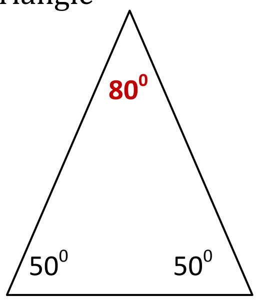
$$\frac{5.8x}{5.8} = \frac{440.92}{5.8}$$

x = 76.0 m width of the river

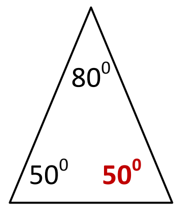
#3. One triangle has two 50^0 angles. Another triangle has a 50^0 angle and an 80^0 angle. Could the triangles be similar? Explain.

Answer:

One Triangle



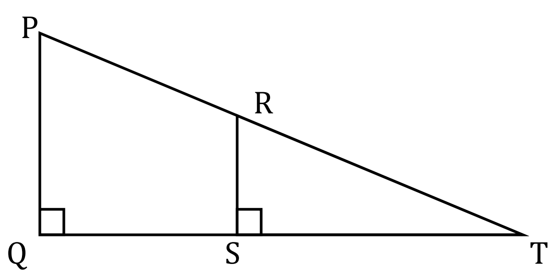
Another Triangle



If you find the missing angle in each triangle you will see they have the same three angles, therefore they are similar.

NOTE: With triangles all you need is to show that the three angles are congruent. In fact, knowing two angles are congruent means the third angle is also congruent. So, having two angles equal in a triangle is enough to prove they are similar.

#4. Use the diagram below to answer the following questions.



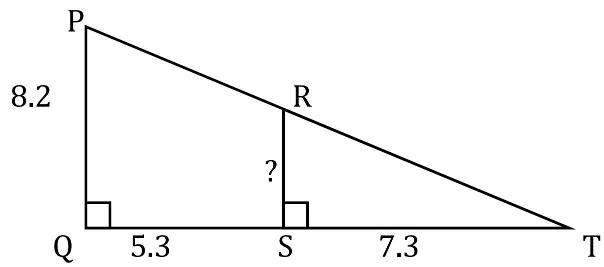
a). Which two triangles are similar? How do you know?

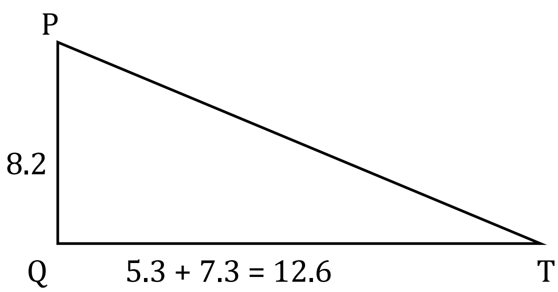
Answer:

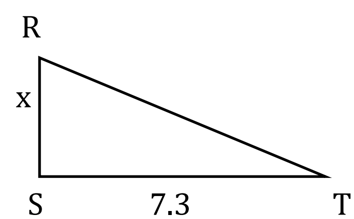
a). ΔPQR is similar to ΔRST because both have the same angle T and both have a 90^0 angle. Having two angles proves there are three angles the same - therefore they are similar.

b). If PQ = 8.2 cm, QS = 5.3 cm and ST = 7.3 cm, find the length of RS.

Answer: Fill in everything you know on the picture. Then separate it into 2 different triangles.







$\frac{PQ}{RS} = \frac{QT}{ST}$ $\frac{8.2}{x} = \frac{12.6}{7.3}$ (8.2)(7.3) = 12.6 x

$\frac{59.86}{12.6} = \frac{12.6\,x}{12.6}$ x = 4.8 cm

7.5 Reflections and Line Symmetry

Taj Mahal is a famous example of symmetry in architecture.

Many parts of the building and grounds were designed and built to be perfectly symmetrical.

Symmetry creates a sense of balance.

Line symmetry

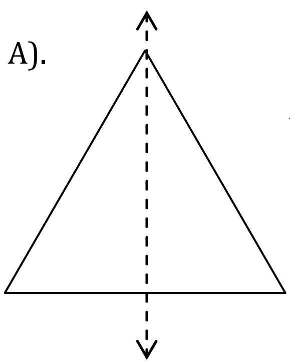
- a figure is divided into 2 congruent parts using a line of symmetry (mirror image)
- one half of the figure is reflected exactly onto the other half
- a figure may have more than one line of symmetry

The line of symmetry (also called line of reflection) can be:

- vertical
- horizontal
- oblique

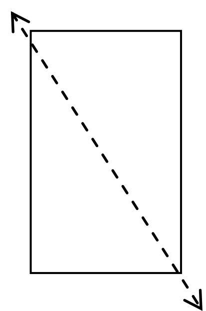
Is the dashed line in each figure a line of symmetry? Explain.

A).



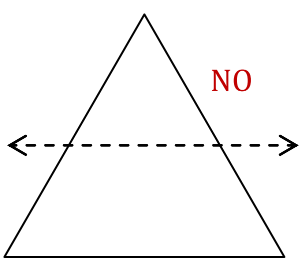
YES

B).



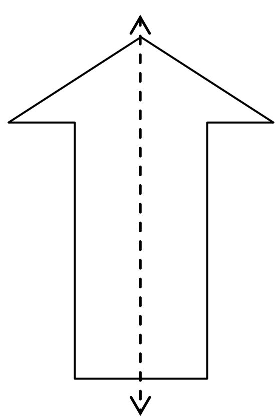
NO

C).



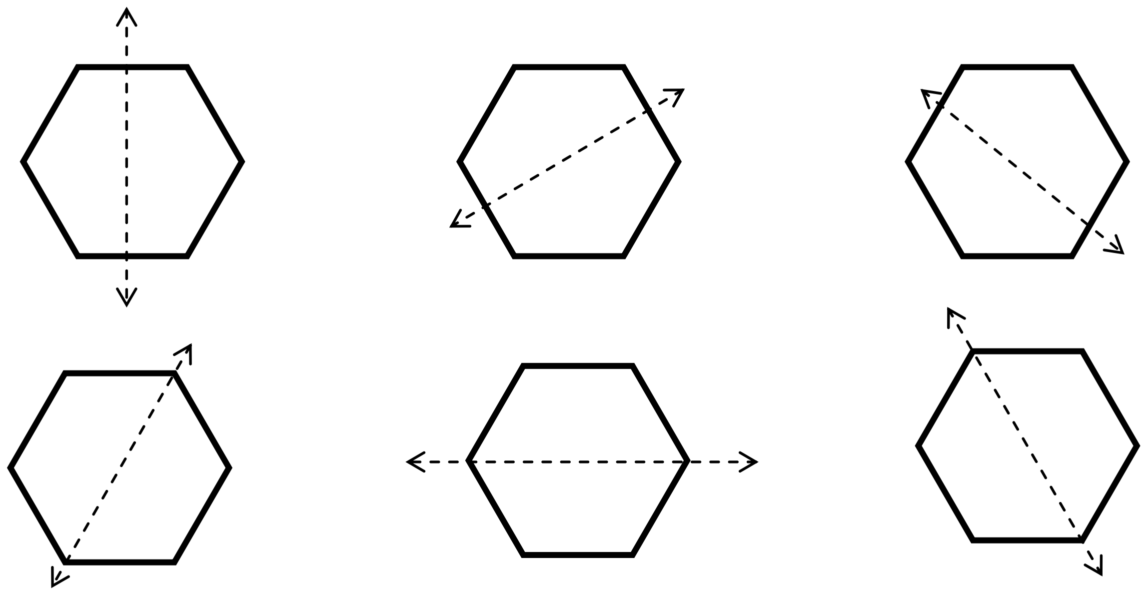
NO

D).



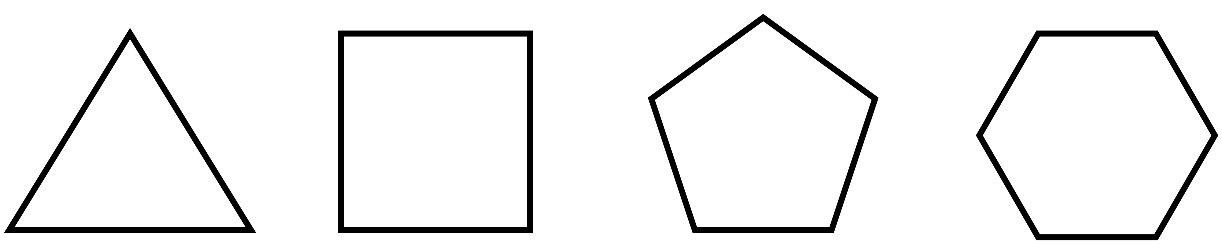
YES

Is each a line of symmetry for the hexagon? YES



Can anymore lines of symmetry be drawn for a hexagon? NO

Investigate the lines of symmetry for regular polygons.

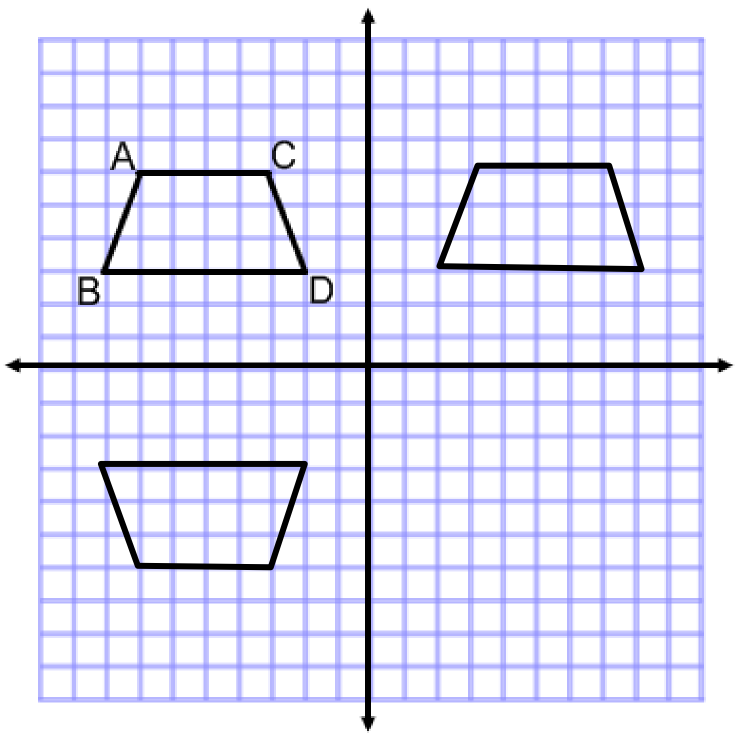


Number of Sides	Number of Lines of Symmetry
3	3
4	4
5	5
6	6
n	n

Make a general statement describing the relationship between the number of sides and the number of lines of symmetry that can be drawn in a polygon.

The number of lines of symmetry is equal to the number of sides in a regular polygon.

Reflecting on the Cartesian Plane



Reflect across the x-axis

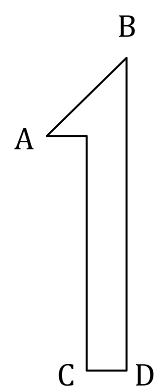
Point	Image
A (-7, 6)	A'(-7, -6)
B (-8, 3)	B'(-8, -3)
C (-3, 6)	C'(-3, -6)
D (-2, 3)	D'(-2, -3)

Reflect across the y-axis

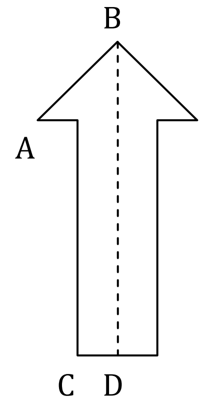
Point	Image
A (-7, 6)	A'(7, 6)
B (-8, 3)	B'(8, 3)
C (-3, 6)	C'(3, 6)
D (-2, 3)	D'(2, 3)

This figure represents half of a shape. Create the final shape by constructing the missing half, use each case below:

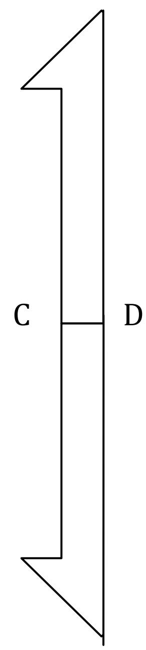
(a). Line of symmetry is BD
(b). Line of symmetry is CD
(c). Line of symmetry is AB



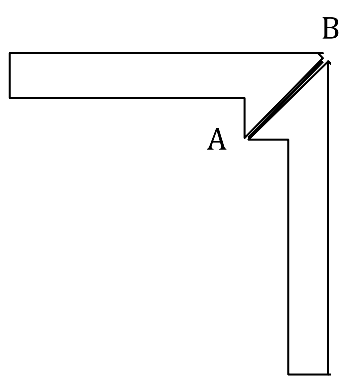
a). Line of symmetry is BD



(b). Line of symmetry is CD



(c). Line of symmetry is AB



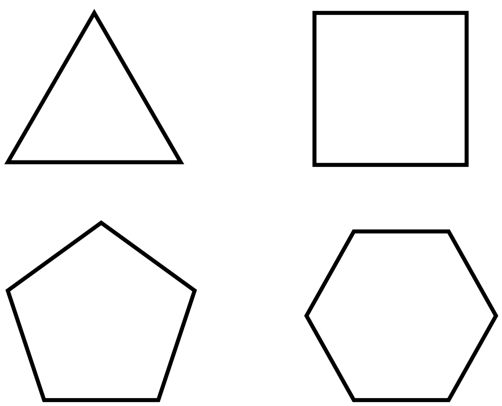
Section 7.6 Rotations and Rotational Symmetry

Rotational Symmetry A figure has rotational symmetry if it can be turned around its center to match itself in less than a 360° turn.

The number of times in one complete turn that a figure matches itself is referred to as: __________

- Order of Rotational Symmetry OR
- Degree of Rotational Symmetry

Use the drawings below to help you determine the order or degree of rotational symmetry for each of the regular polygons.



Number of Sides	Degree or Order of Rotational Symmetry
3	3
4	4
5	5
6	6
n	n

Make a general statement describing the relationship between the number of sides and the degree OR order of rotational symmetry in regular polygons.

- The degree or order of rotational symmetry is equal to the number of sides in a regular polygon.

Angle of Rotational Symmetry

- the minimum angle required for a shape to rotate and coincide with itself is:

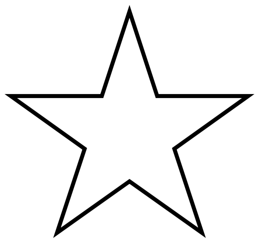
$$\frac{360^\circ}{\text{the order of rotation}}.$$

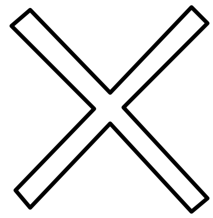
Polygon Summary

Number of Sides	Degree or Order of Rotational Symmetry	Angle of Rotational Symmetry
3	3	120^0
4	4	90^0
5	5	72^0
6	6	60^0

Try these!
What is the Order of Rotational Symmetry?   What is the Angle of Rotational Symmetry?

a). 

b). 

c). 

Answers:

a) Order = 2
Angle = 180^0

b) Order = 5
Angle = 72^0

c) Order = 4
Angle = 90^0

What do you think the order of symmetry is for a circle? INFINITE

What if you know the angle of rotational symmetry and you are asked to find the order of rotational symmetry ?

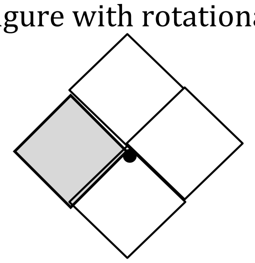
- $\dfrac{360^0}{\text{Angle of rotational symmetry}}$

Examples:
What is the order of rotational symmetry for each angle of rotation symmetry?

A) 90^o Order = $\dfrac{360}{90}$ = 4

B) 120^o Order = $\dfrac{360}{120}$ = 3

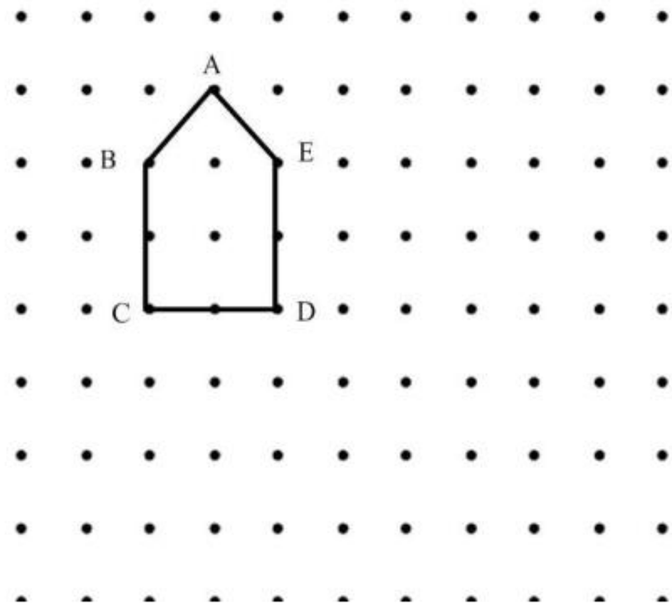
You can create your own figure with rotational symmetry by rotating a shape about a vertex.



Rotations Continued

Example 1:

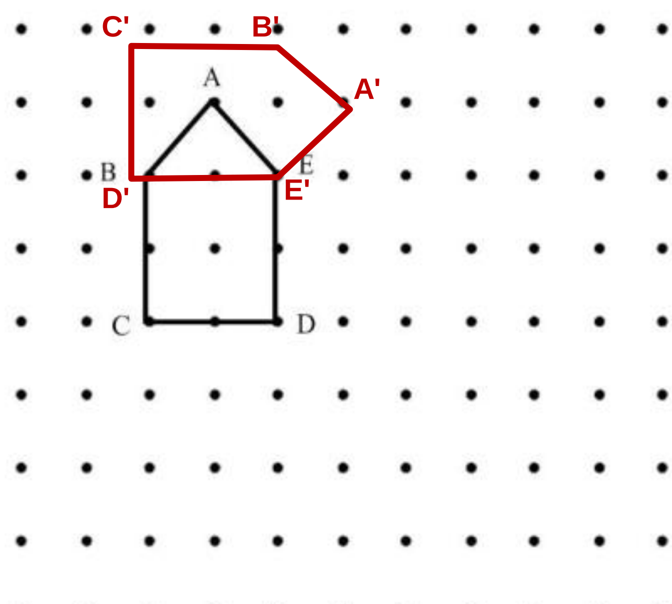
Rotate pentagon ABCDE
90º clockwise about vertex E.
Draw the rotation image.



Answer

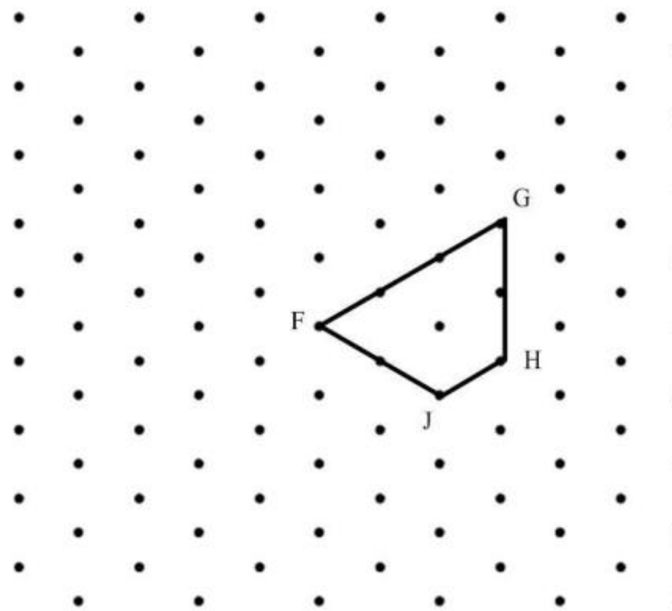
Example 1:

Rotate pentagon ABCDE
90º clockwise about vertex E.
Draw the rotation image.



Example 2:

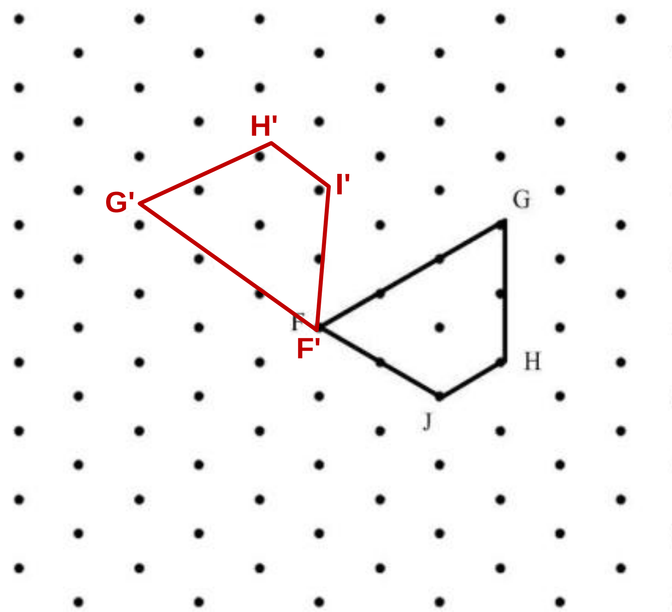
Rotate trapezoid FGHJ 120º counterclockwise about vertex F.
Draw the rotation image.



Answer

Example 2:

Rotate trapezoid FGHJ 120º counterclockwise about vertex F.
Draw the rotation image



Example 3:

a) Rotate rectangle ABCD:

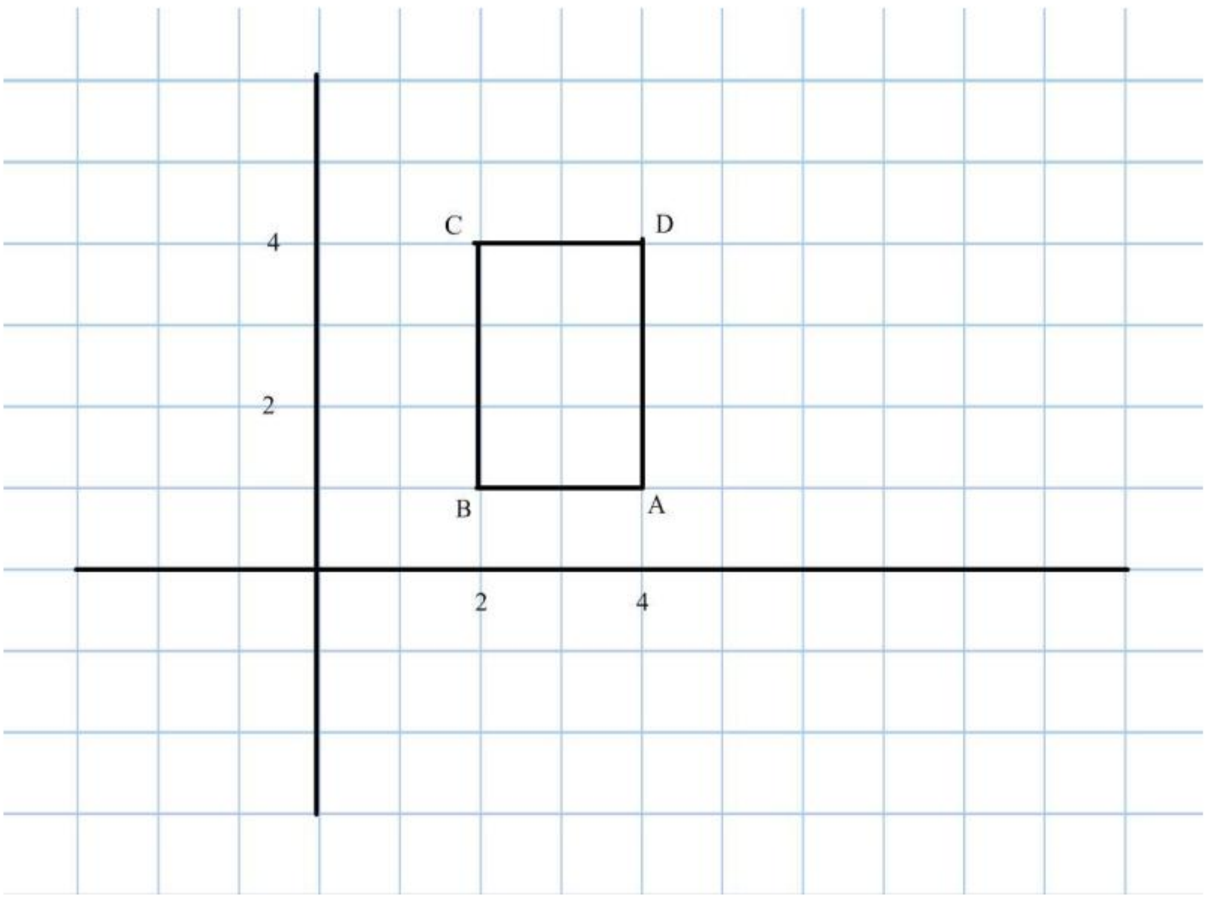
i) 90° clockwise about vertex A

ii) 180° clockwise about vertex A.

iii) 270° clockwise about vertex A.

Draw and label each rotation image.

b) Look at the shape formed by the rectangle and all its images.
Identify any rotational symmetry in this shape.



Answer

Example 3:

a) Rotate rectangle ABCD:

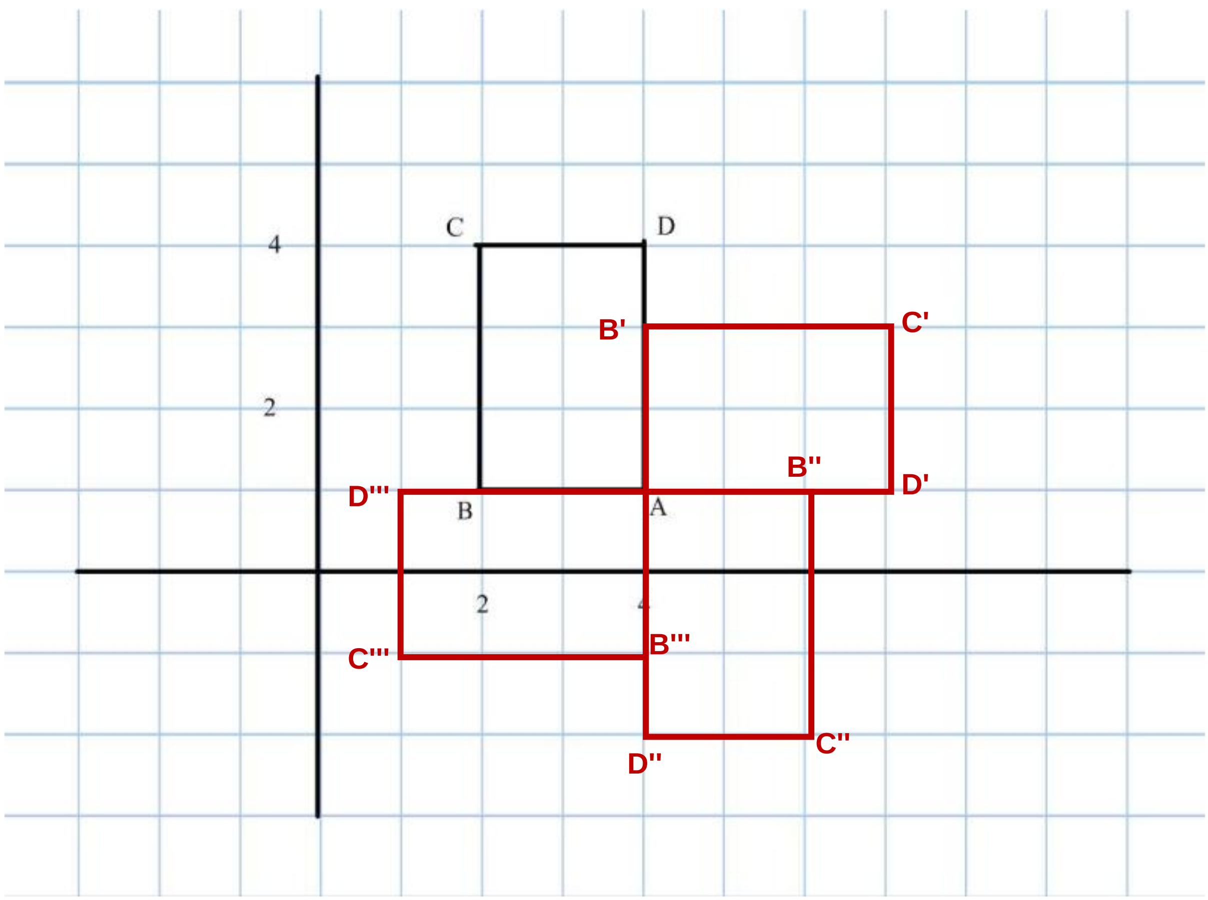
i) 90^{o} clockwise about vertex A

ii) 180^{o} clockwise about vertex A.

iii) 270^{o} clockwise about vertex A.

Draw and label each rotation image.

b) Look at the shape formed by the rectangle and all its images.
Identify any rotational symmetry in this shape.



b). This new image has a rotational symmetry of 4.

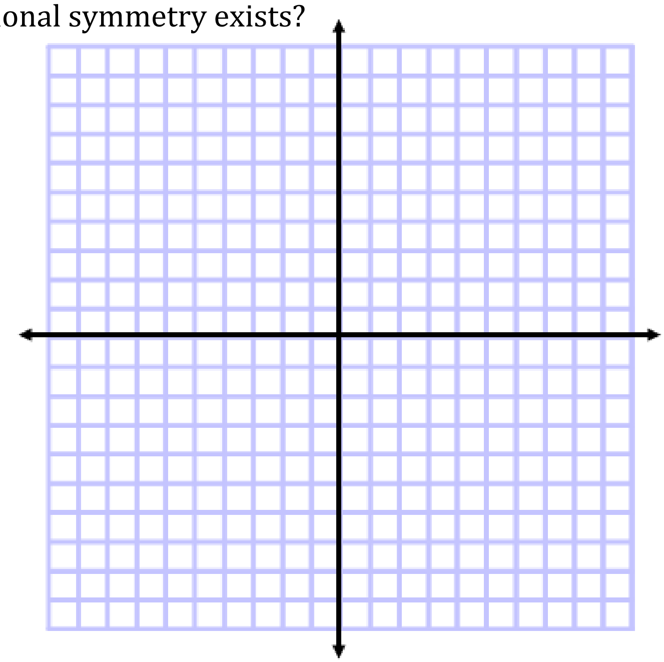
Sec 7.7 Identifying Types of Symmetry on the Cartesian Plane **Gr. 9 Math**

We will be completing translations, reflections and rotations to an image to see if it has reflectional or rotational symmetry.

Example 1:
Draw rectangle ABCD after each transformation. Write the coordinates of each new vertex. Describe whether or not reflectional or rotational symmetry exists?

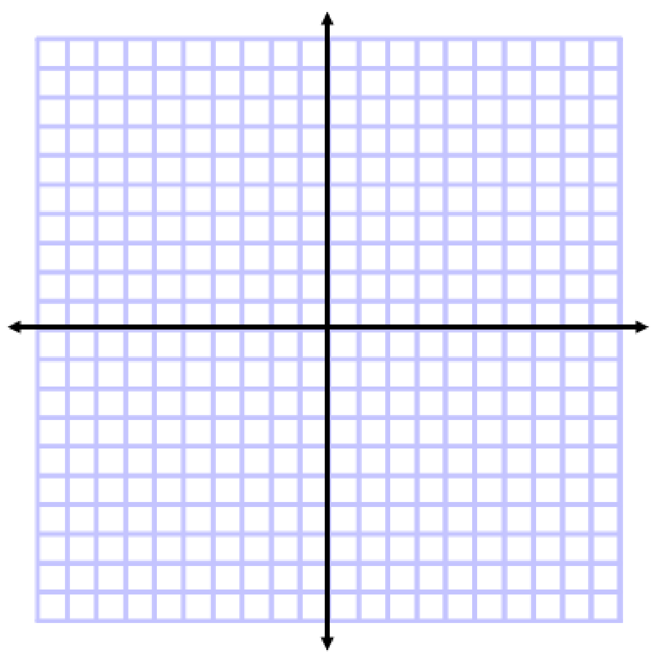
a). a rotation of 180^0 about the origin

Point	**Image**
A(-2,3)	
B(4,3)	
C(4,0)	
D(-2, 0)	



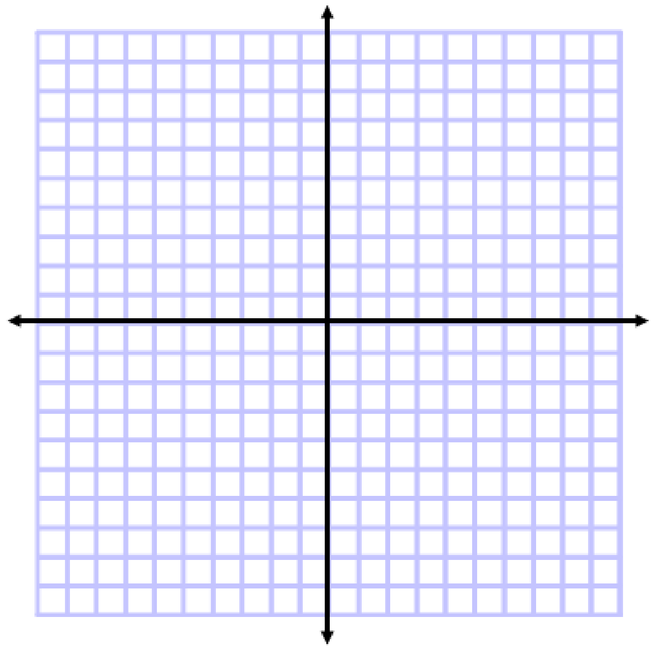
b). a reflection along the x-axis

Point	**Image**
A(-2,3)	
B(4,3)	
C(4,0)	
D(-2, 0)	



c). a translation 3 units right and 1 unit down

Point	Image
A(-2,3)	
B(4,3)	
C(4,0)	
D(-2, 0)	



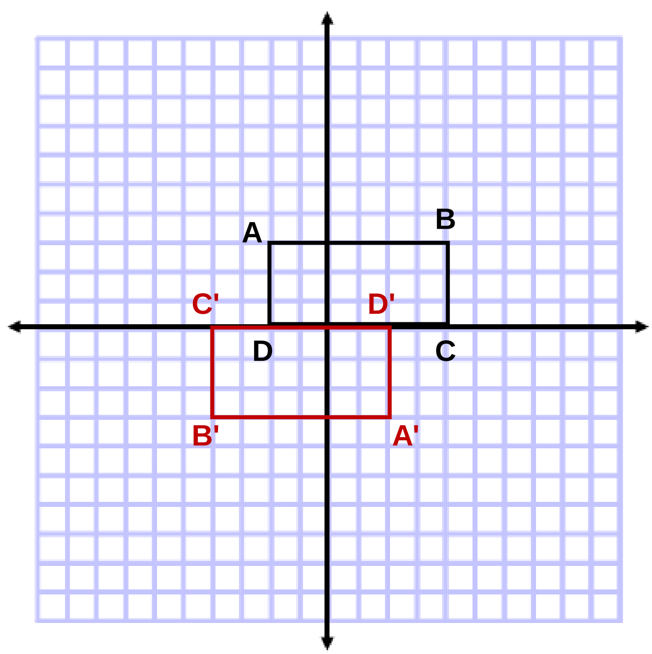
Answer

Example 1:

Draw rectangle ABCD after each transformation. Write the coordinates of each new vertex. Describe whether or not reflectional or rotational symmetry exists?

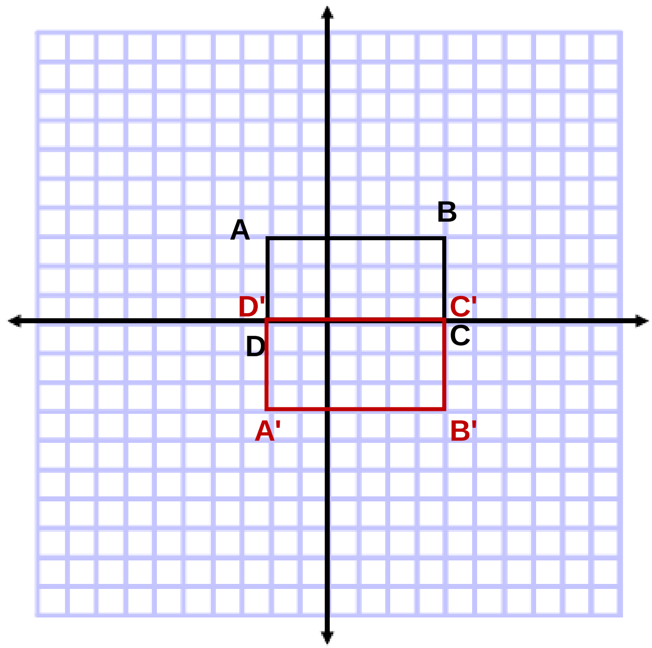
a). a rotation of 180^0 about the origin

Point	Image
A(-2, 3)	A' (2, -3)
B(4,3)	B' (-4, -3)
C(4, 0)	C' (-4, 0)
D(-2, 0)	D' (2, 0)



The octagon that is formed has NO line symmetry but has rotational symmetry about the origin. The octagon has an order of 2.

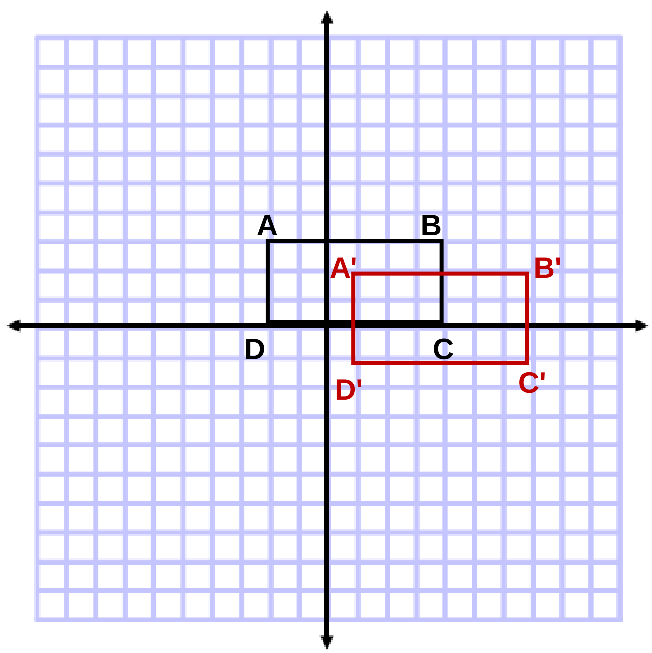
b). a reflection along the x-axis



Point	Image
A(-2, 3)	A' (-2, -3)
B(4,3)	B' (4, -3)
C(4, 0)	C' (4, 0)
D(-2, 0)	D' (2, 0)

This creates a square so it has rotational symmetry of order 4. It also has line symmetry. Four lines can be drawn.

c). a translation 3 units right and 1 unit down



Point	Image
A(-2, 3)	A' (1, 2)
B(4,3)	B' (7, 2)
C(4, 0)	C' (7, -1)
D(-2, 0)	D' (1, -1)

The new octagon does have rotational symmetry of order 2 but it does NOT have line symmetry.