### Grade 9: Unit 5- Polynomials

# **Section 5.1 Modeling Polynomials**

## **Polynomial:**

- an algebraic **expression** that contains one term or a sum of terms.
- The term(s) may contain variables (which will have whole number exponents).
- And a term may be a number.

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3x + 1 → is a polynomial. It contains a variable (whose exponent is 1) and numbers. Since it is an expression, there is no equal sign.
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- → This polynomial has two **terms**. A term is a number, or a variable, or the product of numbers and variables. Terms are separated by + or . Therefore, 3x, is one term and 1, is another term.
- → In the term, 3x, the 3 is called the numerical coefficient. This is the number in front of the variable, it's the numerical factor of a term. X is called the variable.
- → 1 is called the constant term. There is no variable attached to this number. It is the number in the expression that does not change.

# Note:

An algebraic expression that contains a term with a variable in the denominator,

such as  $\frac{3}{n}$ , or the square root of a variable, such as  $\sqrt{n}$ , **is not a polynomial**.

# **Types of Polynomials**

We can classify a polynomial by the numbers of terms it has. Polynomials with 1, 2, or 3 terms have special names.

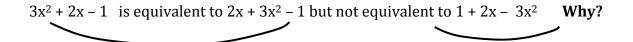
A **monomial** has 1 term; for example: 5x, 9,  $-2p^2$ A **binomial** has 2 terms; for example: 2c - 5,  $2m^2 + 3m$ , x + yA **trinomial** has 3 terms; for example:  $2h^2 - 6h + 4$ , x + y + z Example: Identify (i) the variable

- (ii) the number of terms
- (iii) the numerical coefficient(s)
- (iv) the constant term and
- (v) the type of polynomial

a). $3x^2 + 2x - 1$	(i) x	b). 6xy – x <sup>3</sup>	(i) x
	(ii) 3		(ii) 2
	(iii) 3 and 2		(iii) 6 and −1
	(iv) −1		(iv) none
	(v) trinomial		(v) binomial

c). xy + 6 - z + 2x<sup>2</sup>
(i) x, y and z
(ii) 4
(iii) +1, -1 and 2.
(iv) + 6
(v) just a polynomial
(more than 3 terms does not have a special name).

**Equivalent Polynomials** - are polynomials that have exactly the same terms, but the terms could be in a different order.



Both these polynomials have +3 with x<sup>2</sup> +2 with x and a constant term of – 1 This polynomial is different because it has – 3 with  $x^2$  and a constant term of + 1

### **Degree of a Polynomial**

**Degree:** The term with the greatest exponent.

### Rules for determining the degree:

• The **degree of a monomial** is the sum of the exponents of its variables.

Monomial	Degree
4x <sup>2</sup>	2
9ab	2

• The **degree of a polynomial with one variable** is the highest power of the variable in any one term.

Polynomials	Degree
$6x^2 + 3x$	2
$7 + x^2 - 1$	2

**Example:** Name the coefficients, degree and the constant term of each polynomial.

**A:** -3x<sup>2</sup> + 4x - 5 **B:** 3 + 2ab - b **C**: -6 - 5x

### Solution:

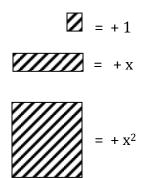
A:Coefficients: -3 and 4<br/>Degree: 2B:Coefficients: -1 and 2<br/>Degree: 2C:Coefficients: -5<br/>Degree: 1Constant Term: -5Constant Term: 3C:Constant Term: -6

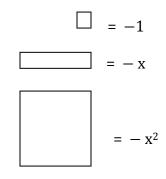
# **Modeling Polynomials**

In algebra we use algebra tiles to model integers and variables.

# Shaded tiles represent **positive** tiles

Non-shaded tiles represent **negative** tiles





Colors can also be used to represent a tile.

### **IN YOUR TEXTBOOK**:



The variable most commonly used is X, however, any variable can be used.

### **REAL ALGEBRA TILES:**

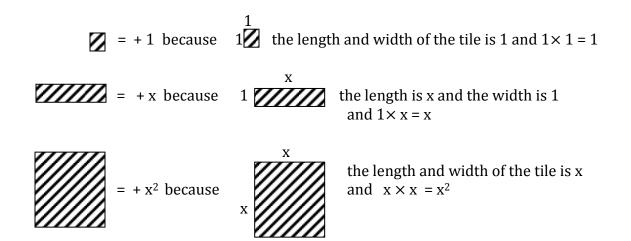
Green and Red is Positive

White is Negative



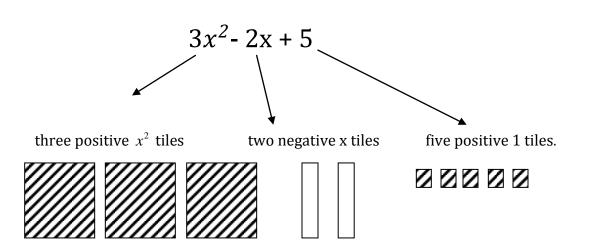
\*\*\*\* To be clear in your notes: Shaded is Positive Unshaded is Negative

Algebra tiles get their name from the area of their tiles. Remember length × width = area

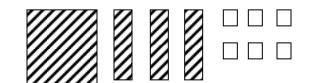


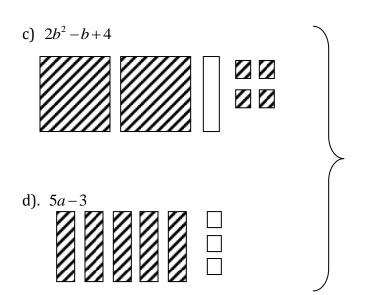
# **Examples**:

- 1. Use algebra tiles to model each expression.
- a).  $3x^2 2x + 5$



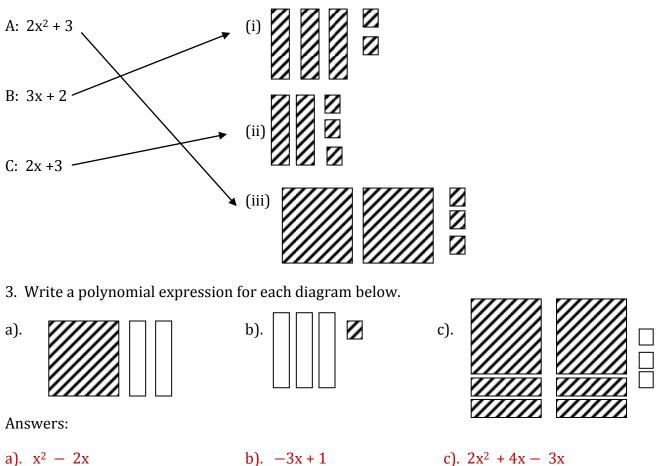
b).  $x^2 + 3x - 6$ 





Remember any variable can be used instead of x.

2. Match the following polynomials to the appropriate diagram.



A polynomial should be written in **descending order**. This means the exponent of the variable should decrease from left to right.

Ex: The polynomial  $2k - 4k^2 + 7$  is properly written as  $-4k^2 + 2k + 7$  in descending order.

4. Rearrange the following polynomials in descending order.

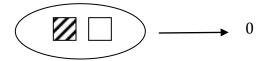
a).  $-2p + 4p^2 - 9$  b).  $5x^2 + 7 - 8x$  c).  $33 + 90c + 100c^2$ 

Answers:

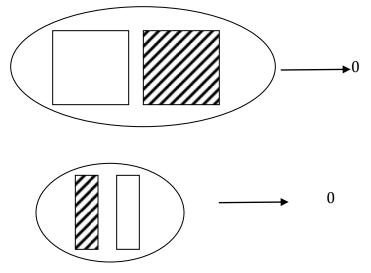
a).  $4p^2 - 2p - 9$  b).  $5x^2 - 8x + 7$  c).  $100c^2 + 90c + 33$ 

# Section 5.2 Like Terms and Unlike Terms

When you worked with integers, a +1 tile and a -1 tile formed a **zero pair**.



The same applies for the x and  $x^2$  tiles.

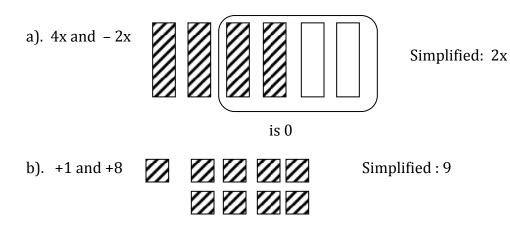


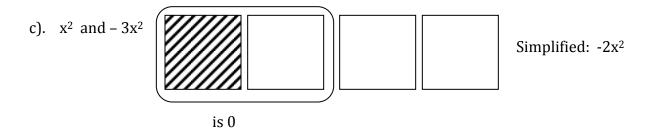
Any two **opposite colored tiles** of the **same size** has a sum of **zero**. We can combine these tiles because they are **like terms**.

Like Terms – terms that have the same variable, raised to the same exponent.

Examples: a). 4x and – 2x b). +1 and +8 c). x<sup>2</sup> and – 3x<sup>2</sup>

Like terms can be combined or simplified. Sketch the tiles above and cancel the zero pairs where possible, to simplify the polynomial.

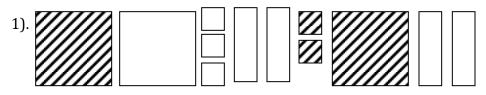




**Unlike Terms** - terms which contain different variables entirely or are the same variable raised to different exponents.

Examples: a). x + yb). 2x + 3c).  $4x + 2x^2$  These are simplified as much as possible already because they don't contain any like terms.

Examples: Write a simplified expression for the algebra tiles below.

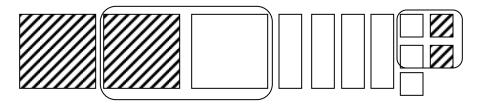


You can rearrange the tiles so you have like terms next to each other.



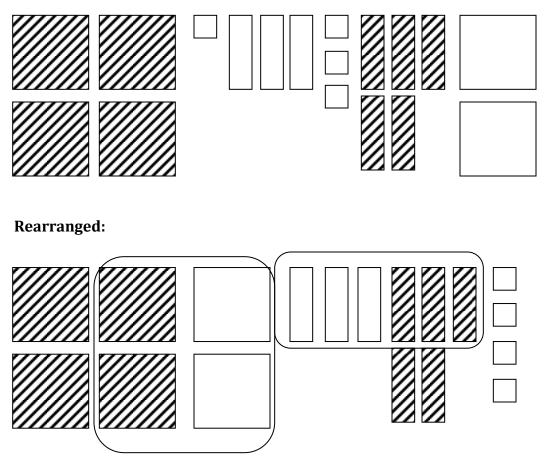
These tiles represent the polynomial  $2x^2 - x^2 - 4x - 3 + 2$ 

Cancel zero pairs to simplify.



Answer:  $x^2 - 4x - 1$ 

Therefore, without tiles, the polynomial  $2x^2 - x^2 - 4x - 3 + 2$  simplifies to  $x^2 - 4x - 1$ 



2). Sketch the simplified expression using algebra tiles for:  $4n^2 - 1 - 3n - 3 + 5n - 2n^2$ 

**Simplified answer:**  $2n^2 + 2n - 4$ 

Therefore the polynomial  $4n^2 - 1 - 3n - 3 + 5n - 2n^2$  simplified to  $2n^2 + 2n - 4$ .

Can you see how to simplify like terms without using tiles?

$$4n^{2}-1-3n-3+5n-2n^{2} \text{ means } 4n^{2} \text{ and } -2n^{2} = 2n^{2}$$

$$-1-3n-3+5n \text{ means } -3n \text{ and } 5n = 2n$$

$$-1-3 \text{ means } -1 \text{ and } -3 = -4$$

$$Answer: 2n^{2}+2n-4$$

- 3). Simplify each polynomial without using tiles.
  - A: 3x + 5xB: -13a 10a= 8x= -23aC: 16n + n 17nD: -j + 7k 3j= 0= -4j + 7kE: 8a 2b 6a 3bF: -q + 7q + 11n + 11p 8q
  - = 2a 5b = -2q + 11n + 11p
- 4. Wayne was asked to write an expression equivalent to 2x 7 4x + 8.

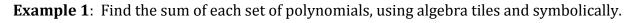
His solution was:

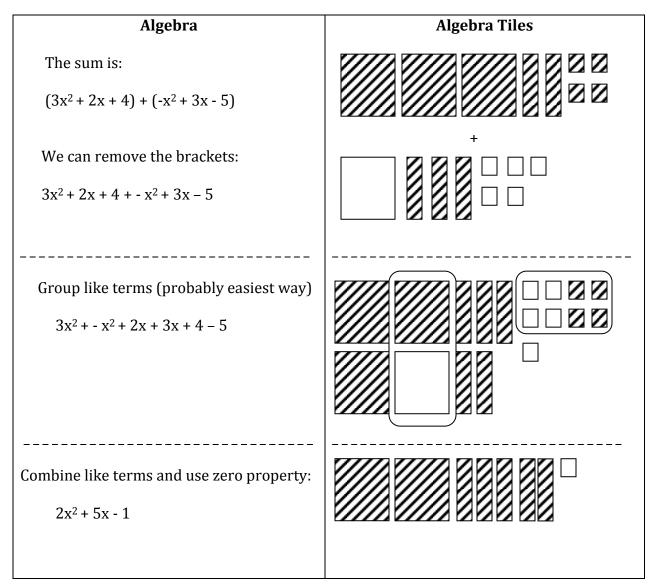
2x - 7 - 4x + 8= 2x - 4x - 7 + 8

- a). What errors did he make?
- → When he combined 2x 4x, he said the answer was 2x and it should have been 2x.
   → When he combined 7 + 8, he said the answer was 1 and it should have been +1.
- b). Show the correct simplification.
  - 2x 7 4x + 8= 2x 4x 7 + 8= -2x + 1

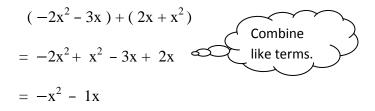
## Section 5.3 Adding Polynomials

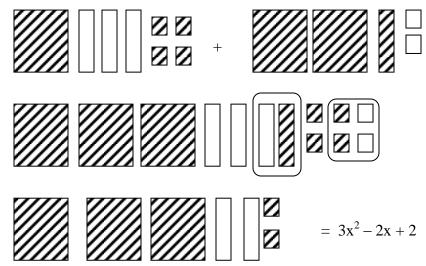
To add or subtract separate polynomials, you just need to combine like terms.





**Example 2:** Add symbolically (using algebra)





**Example 3:** Add: using algebra tiles. Write your answer using tiles and symbolically.

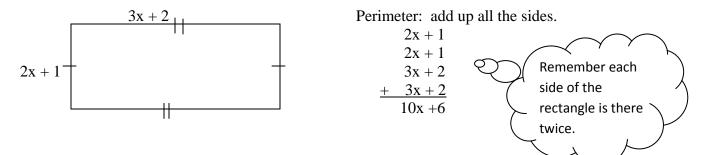
You can add polynomials horizontally and vertically.

Try:  $(7n + 14) + (-6n^{2} + n - 6)$ Horizontally  $7n + 14 + -6n^{2} + n - 6$   $= -6n^{2} + n + 7n + 14 - 6$   $= -6n^{2} + 8n + 8$ Horizontally just group like terms and simplify Vertically  $-6n^{2} + n - 6$   $+ \frac{7n + 14}{-6n^{2} + 8n + 8}$ Vertically line up like terms and simplify

**Example 4**: Add  $(2x^2 + 3x - 2) + (-x^2 + 7x - 3)$  both horizontally and vertically.

#### Horizontally $2x^{2} + 3x - 2 + -x^{2} + 7x - 3$ $2x^{2} + -x^{2} + 3x + 7x - 2 - 3$ $= x^{2} + 10x - 5$ Vertically $2x^{2} + 3x - 2$ $+ -x^{2} + 7x - 3$ $x^{2} + 10x - 5$

**Example 5:** Write a polynomial for the **perimeter** of this rectangle.



**Example 6:** Adding polynomials in two variables

Add: 
$$(2a^{2} + a - 3b - 7ab + 3b^{2}) + (-4b^{2} + 3ab + 6b - 5a + 5a^{2})$$
  
=  $2a^{2} + a - 3b - 7ab + 3b^{2} + -4b^{2} + 3ab + 6b - 5a + 5a^{2}$   
=  $2a^{2} + 5a^{2} + 3b^{2} - 4b^{2} + a - 5a - 3b + 6b - 7ab + 3ab$   
=  $7a^{2} - b^{2} - 4a + 3b - 4ab$ 

### **Question:**

A student added  $(4x^2 - 8x + 1) + (2x^2 - 6x - 2)$  as follows.  $(4x^2 - 8x + 1) + (2x^2 - 6x - 2)$   $= 4x^2 - 8x + 1 + 2x^2 - 6x - 2$   $= 4x^2 + 2x^2 - 8x - 6x + 1 - 2$  $= 6x^2 - 2x - 1$ 

(i) Is the students work correct?

(ii) If not, explain where the student made any errors and write the correct answer.

Answer: This student is not correct, they made a mistake combining their x-term. - 8x - 6x = -14x not - 2x

Correct answer:  $6x^2 - 14x - 1$ 

### Section 5.4 Subtracting Polynomials

Remember from earlier this year the word "opposite".

What is the <b>opposite of 2.4</b> ?	What is the <b>opposite of</b> – <b>10</b> ?		
Answer: $-2.4$	Answer: 10		

By definition, opposite numbers have a sum of zero. The same idea applies to polynomials. **Opposite polynomials will have a sum of zero**.

What is the <b>opposite of 2x</b> ?	What is the <b>opposite of</b> – $x^2$ ?
Answer: – 2x	Answer: x <sup>2</sup>

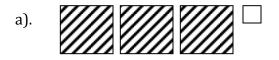
**Example 1**: What is the opposite of each polynomial listed below?a). -5xb). 11c).  $-24x^4$ Getting the opposite of a<br/>monomial is just like<br/>getting the opposite of a #.Answers: a). 5xb). -11c).  $24x^4$ 

How do you think you will get the opposite of a binomial or trinomial?

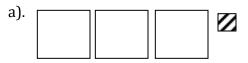
d). 2x + 3Answer: -2x - 3e).  $4x^2 - 7x + 3$ Answer:  $-4x^2 + 7x - 3$ 

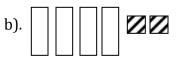
f).  $(-2xy - 2y^2 + 3x^2)$  Answer:  $2xy + 2y^2 - 3x^2$ 

**Example 2**: Sketch the opposite of the polynomial using algebra tiles.



Answers:







When subtracting polynomials you must remember to **ADD THE OPPOSITE** of every term in the polynomial first, then combine like terms.

We will be subtracting polynomials symbolically and using algebra tiles.

**Example 3**: Subtract using algebra, then simplify.

a).  $(3x^2 - 6x + 4) - (7x^2 + 3x - 2)$ We must add the opposite of every term in this polynomial. The first

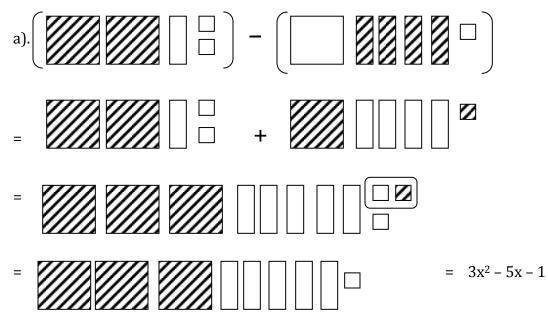
polynomial does not change.

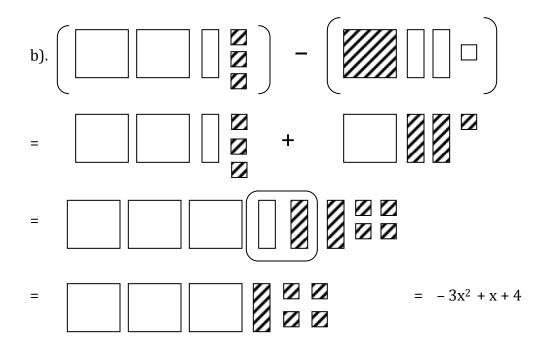
=  $(3x^2 - 6x + 4) + (-7x^2 - 3x + 2)$  Now you are back to adding polynomials.

- $= 3x^2 7x^2 3x 6x + 4 + 2$
- $= -4x^2 9x + 6$

b). 
$$(-2a^{2} + a - 1) - (a^{2} - 3a + 2)$$
  
=  $(-2a^{2} + a - 1) + (-a^{2} + 3a - 2)$   
=  $-2a^{2} - a^{2} + a + 3a - 1 - 2$   
=  $-3a^{2} + 4a - 3$ 

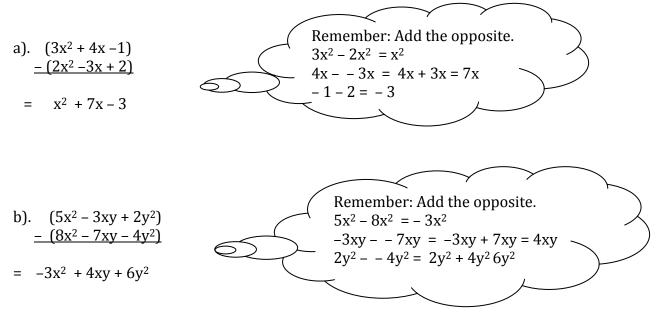
**Example 4**: Subtract using algebra tiles, then simplify.





Just like with adding, we can subtract polynomials horizontally and vertically.

**Example 5**: Subtract vertically.



1	Algebra	Algebra Tiles
	(3x <sup>2</sup> – 4x) – (2x <sup>2</sup> – x)	
	(3x <sup>2</sup> – 4x) + (–2x <sup>2</sup> + x)	
	$3x^2 - 2x^2 - 4x + x$	
	x <sup>2</sup> – 3x	

**Example 6**: Subtract using algebra and algebra tiles.

# Example 7:

A student subtracted like this:  $(2y^2 - 3y + 5) - (y^2 + 5y - 2)$   $= 2y^2 - 3y + 5 - y^2 + 5y - 2$  $= 2y^2 - y^2 - 3y + 5y + 5 - 2$ 

 $= y^2 - 2y + 3$ 

- (i) Explain why the solution is incorrect.
- (ii) What is the correct answer? Show your work.

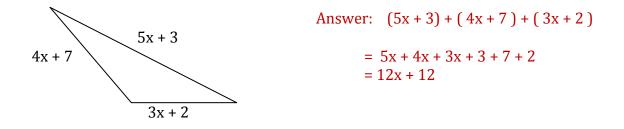
### Answer:

(i) They added the opposite incorrectly. They only got the opposite of  $y^2$ , when they should have gotten the opposite of every term in the polynomial, including the opposite of 5y and -2.

(ii) 
$$(2y^2 - 3y + 5) - (y^2 + 5y - 2)$$
  
=  $2y^2 - 3y + 5 - y^2 - 5y + 2$   
=  $2y^2 - y^2 - 3y - 5y + 5 + 2$   
=  $y^2 - 8y + 7$ 

# **Application of Adding and Subtracting**

1a. Write a simplified expression for the perimeter of the triangle.



b. If the value of x = 4 cm , what is the perimeter of the triangle?

2. Subtract  $2x^2 + 2x + 5$  from  $5x^2 - 7x + 4$ 

Means:  $(5x^2 - 7x + 4) - (2x^2 + 2x + 5)$ =  $5x^2 - 7x + 4 - 2x^2 - 2x - 5$ =  $3x^2 - 9x - 1$ 

3. Subtract the sum of a + b and 2a – b from 4a – 4b.

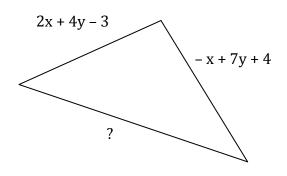
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Sum: (a + b) + (2a - b) = 3a
Answer: (4a - 4b) - (3a)
= 4a - 4b - 3a
= a - 4b
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4. Write a monomial that describes the perimeter.



Perimeter: 4x + x + 2x + x + 2x + 2x = 12x

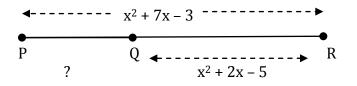
5. Find the missing side if the Perimeter is 5x + 3y - 2.



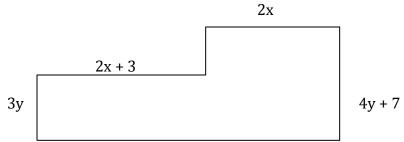
Need sum of given sides first. (2x + 4y - 3) + (-x + 7y + 4)= x + 11y + 1

Subtract sum of sides from Perimeter (5x + 3y - 2) - (x + 11y + 1) = 5x + 3y - 2 - x - 11y - 1 = 4x - 8y - 3... is the length of the missing side.

6. Find the length of PQ.

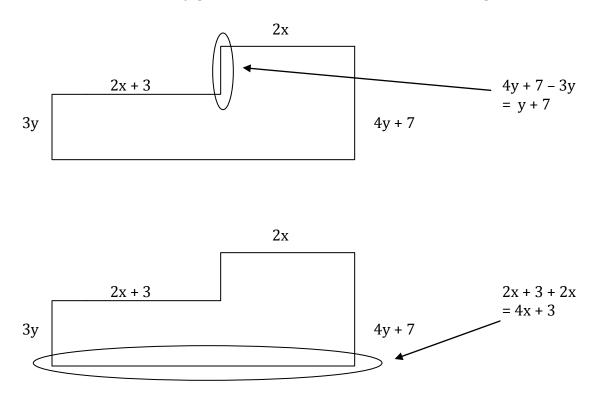


Answer:  $(x^2 + 7x - 3) - (x^2 + 2x - 5)$ =  $x^2 + 7x - 3 - x^2 - 2x + 5$ = 5x + 2 7a. Write a simplified expression for the perimeter.



Perimeter means to add up all the sides. How many sides does this shape have?

6 sides, but we are only given 4 so we need find the other 2 missing sides.



Perimeter = (3y) + (2x + 3) + (y + 7) + (2x) + (4y + 7) + (4x + 3)= 8y + 8x + 20

b). What is the perimeter if x = 1 cm and y = 2cm? = 8y + 8x + 20= 8(2) + 8(1) + 20 = 16 + 8 + 20 = 44 cm.

### **Multiplying Polynomials**

(Sec 5.5 and Sec 5.6)

Remember:

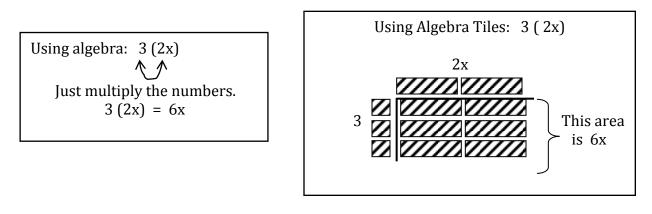
When multiplying or dividing ....

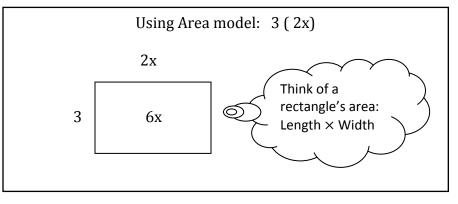
+ and + = + — and + = — — and — = + + and — = —

We will only be multiplying a polynomial by a monomial. The monomial could be a constant term, ex: 3(2x) or 3(2x + 2) or it could contain a variable, ex: 3x (2x) or 3x (2x + 2), etc.

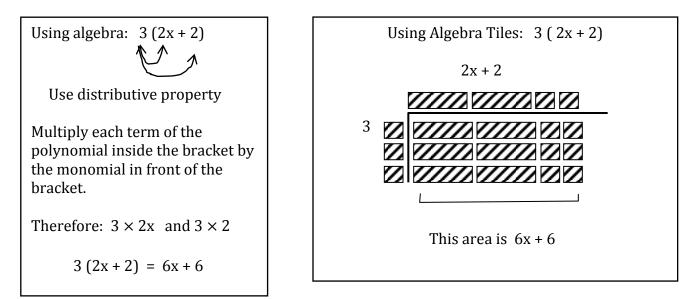
Students will be expected to multiply polynomials using symbolically, using area model and algebra tiles.

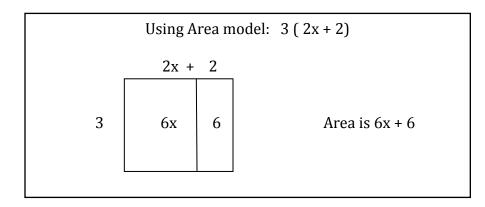
**Example 1**: 3 (2x)



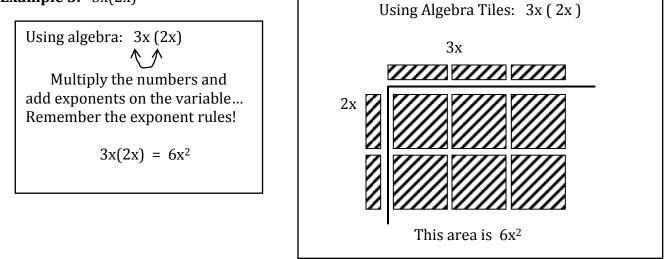


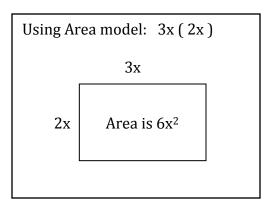
**Example 2:** 3(2x + 2)



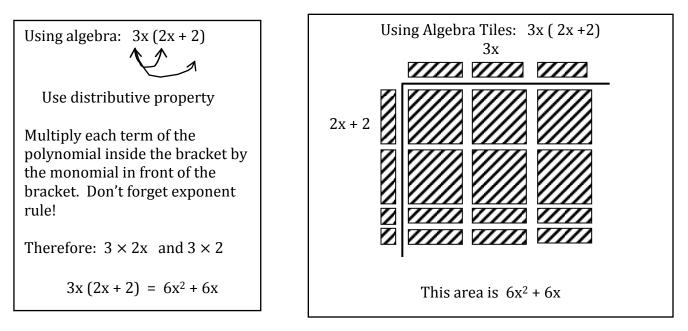


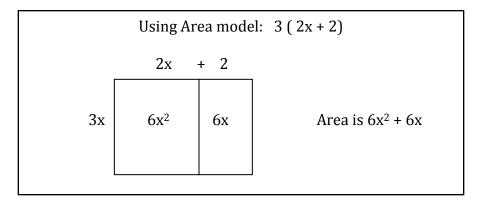
**Example 3:** 3x(2x)





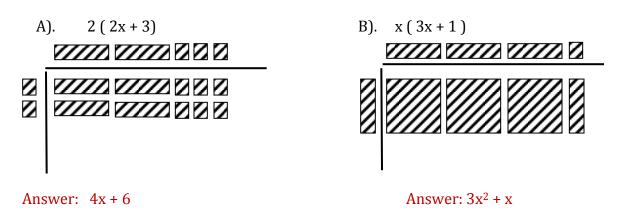
# **Example 4:** 3x(2x + 2)



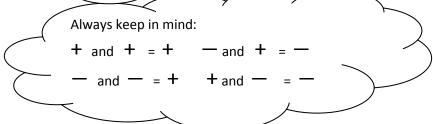


Try These!

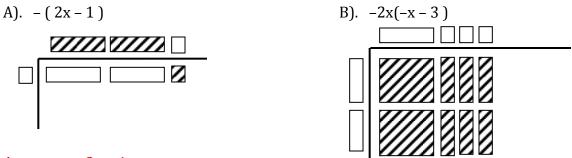
1. Multiply using algebra tiles.



- 2. Multiply using distributive property...using algebra. Careful with signs!
- A). 3(-2m + 4)= -6m + 12B). -4(x + 2)= -4x - 8C).  $-2(-n^2 + 2n - 1)$ =  $2n^2 - 4n + 2$
- 3. How would you sketch negatives with algebra tiles?
- A). 3(-2m + 4)B). -4(x + 2)B). -4(x + 2)Answer: = -6m + 12B). -4(x + 2)B). -4(x + 2)Comparison of the second s



4. Try These using algebra tiles! Check your answer using algebra.



Answer: -2x + 1

Answer:  $2x^2 + 6x$ 

5. Sketch the answer using the area model:  $-2(-n^2 + 2n - 1)$ 

	-n <sup>2</sup>	+ 2n	- 1	
-2	2n <sup>2</sup>	-4n	+ 2	Answer: $= 2n^2 - 4n + 2$

# 6. Multiply using distributive property.

A:	2( x + 10 )	<b>B:</b>	5y( y + 1 )	C:	-10( x + 2 )
	= 2x + 20		$=5y^2+5y$		= -10x - 20
D:	6x(12-x)	<b>E:</b>	3( x – 7 )	F:	-4x(2x-3)
	$=72x-6x^2$		= 3x - 21		$= -8x^2 + 12x$
G:	-6m( m+4 )	Н:	-8(x-5)	I <b>:</b>	3(-8-7x)
	$=-6m^{2}-24m$		= -8x + 40		= -24 - 21x

# **Dividing Polynomials**

Remember:

When multiplying or **dividing** ....

We will only be dividing a polynomial (one or more terms) by a monomial, symbolically, using algebra tiles and area models. The monomial could be a constant term or contain a variable

Ex: 
$$4x^2 \div 2 = \frac{4x^2}{2}$$
 or  $4x^2 \div 2x = \frac{4x^2}{2x}$   $\frac{4x^2 - 8x}{2}$  or  $\frac{4x^2 - 8x}{2x}$ , etc.

### **Dividing Symbolically:**

<u>4x<sup>2</sup></u>	When dividing a monomial by a monomial You	$\underline{4x^2} = 2x^2$
2	just divide the numbers like normal.	2
<u>4x<sup>2</sup></u>	When dividing a monomial by a monomial and	
2x	there is also a variable in the denominator, you	$\underline{4x^2} = 2x$
	must remember the exponent rule. When	2x
	dividing powers with the same base, you	
	subtract exponents. Still divide the numerical	
	coefficients like normal.	
$4x^2 - 8x$	you can rewrite the quotient as a sum of two	$\underline{4x^2} - \underline{8x} = 2x^2 - 4x$
2	fractions and divide like it is two monomials.	2 2
$4x^2 - 8x$	Rewrite the quotient as a sum of two fractions	$\underline{4x^2} - \underline{8x} = 2x - 4$
2x	and divide like it is two monomials. Don't	2x 2x
	forget the exponent rules when there is a variable	
	in the denominator.	

### NOTE:

However many terms are in the numerator, that's how many terms are in your answer. When dividing a trinomial by a monomial, you will have a trinomial answer.

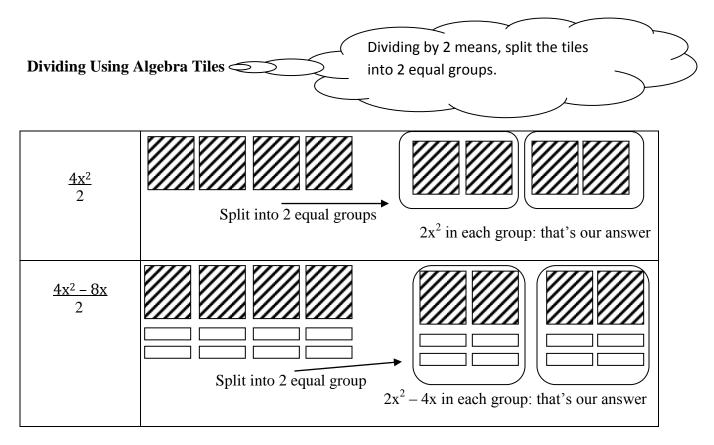
Ex5: 
$$\frac{12m^2 + 6m - 9}{3} = \frac{12m^2}{3} + \frac{6m}{3} - \frac{9}{3} = 4m^2 + 2m - 3$$

Be careful when dividing by negatives!

Ex6: 
$$-3y^2 + 15xy - 21x^2 = -3y^2 + 15xy - 21x^2 = y^2 - 5xy + 7x^2$$

+ and + = + - and + = -- and - = + + and - = -

(Sec 5.5 and Sec 5.6)



## **Dividing Using an Area Model and Algebra Tiles**

a). Find the missing dimension if the area of the rectangle is  $4x^2$  and the length is 2x.

Area Model	Algebra Tiles
$2x  4x^2 \qquad \frac{4x^2}{2x} = 2x$	$ \begin{array}{c} ? \\ \hline  \\ $

b). Find the missing dimension if the area of the rectangle is  $4x^2 - 8x$  and the length is 2x.

