## Grade 9 Mathematics

Unit 2: Powers and Exponent Rules

## Sec 2.1 What is a Power



The entire $2^{5}$ is called a POWER.
$2^{5}=2 \times 2 \times 2 \times 2 \times 2$ written as repeated multiplication.
$2^{5}=32$ written in standard form.

$$
2^{5}=2 \times 2 \times 2 \times 2 \times 2=32
$$

$\begin{array}{ccc}\text { Power } & \begin{array}{c}\text { Repeated } \\ \text { Multiplication }\end{array} & \text { Standard } \\ & \text { Form }\end{array}$

To evaluate a power means to find the answer in standard form.

Are the base and the exponent interchangeable? In other words, does $2^{5}=5^{2}$ ?

$$
2^{5}=2 \times 2 \times 2 \times 2 \times 2=32 \quad 5^{2}=5 \times 5=25
$$

- No, the base and exponent cannot be switched and still be equal.

CHALLENGE!!!! Can you think of one example where the base and exponent can be switched, and the answers are still equal?
$4^{2}$ When you have an exponent of 2, it's called a squared number.


$$
4^{2}=4 \times 4=16
$$

$$
\begin{aligned}
4^{3} & =4 \times 4 \times 4 \\
& =64
\end{aligned}
$$

## The Importance of Brackets

$(-3)^{2} \quad$ The brackets tell us that the base is -3 .

- $(-3)^{2}=(-3) \times(-3)=+9$

When there is an EVEN NUMBER of negatives then the product is POSITIVE.

- $(-3)^{3}=(-3) \times(-3) \times(-3)=-9$

When there is an ODD NUMBER of negatives then the product is NEGATIVE.
$-3^{2} \quad$ There are no brackets so the base is 3 .
The negative applies to the whole expression.

- $-3^{2}=-(3 \times 3)=-9$

Question.

1. Identify the base and evaluate each power.
a). $(-5)^{4}$
b). $-5^{4}$
C). $-(-5)^{4}$
d). $(-5)^{3}$
e). $-5^{3}$
f). $-(-5)^{3}$

Answers:
a). base is ( -5 ), evaluated $=625$
b). base is 5 , evaluated $=-625$
c). base is ( -5 ), evaluated $=-625$
d). base is $(-5)$, evaluated $=-125$
e). base is 5 , evaluated $=-125$
f). base is $(-5)$ evaluated $=125$

Sec 2.2 Powers of Ten and the Exponent Zero
Investigation

| Power | Repeated Multiplication | Standard Form |
| :---: | :---: | :---: |
| $3^{5}$ |  |  |
| $3^{4}$ |  |  |
| $3^{3}$ |  |  |
| $3^{2}$ |  |  |
| $3^{1}$ |  |  |

Look for the patterns in the columns.
The exponent decreases by $\qquad$ each time.

Each time the exponent decreases, standard form in divided by $\qquad$ .

This pattern suggests that $3^{0}=$ $\qquad$ .

A power with exponent 0 is equal to $\qquad$ -.

1a). Complete the table below.

| Power | Repeated Multiplication | Standard Form |
| :---: | :---: | :---: |
| $5^{4}$ |  |  |
| $5^{3}$ |  |  |
| $5^{2}$ |  |  |
| $5^{1}$ |  |  |

b). What is the value of $5^{1}$ ? $\qquad$
c). What is the value of $5^{0}$ ? $\qquad$

## Zero Exponent Rule:

Any base (excluding zero) with the exponent zero is one.
$\mathrm{a}^{0}=1$
where $a \neq 0$

Examples: Remember, any base with the exponent zero is one.

1. Identify the base, then evaluate the answer.
a). $5^{0}$
b). $10^{0}$
c). $(-5)^{0}$
d). $-10^{0}$

## Answers

1a). The base is 5 so $5^{0}=1$
$1 \mathrm{c})$. The base is $-5,(-5)^{0}=1$

1b). The base is 10 , so $10^{0}=1$

1d). The base is 10 , (not -10 ), so $-10^{0}=-1$ BE CAREFUL!
2. Evaluate the following powers. Remember the order of operations!
a). $3+2^{0}$
b). $3^{0}+2^{0}$
c). $(3+2)^{0}$
d). $-3^{0}+2$
e). $-3^{0}+(-2)^{0}$
f). $-(3+2)^{0}$

Answers
2a). $3+2^{0}$
$=3+1$
$=4$
2b). $3^{0}+2^{0}$
= $1+1$
$=2$
2d). $\begin{aligned} & -3^{0}+2 \\ = & -1+2 \\ = & 1\end{aligned}$
2e). $-3^{0}+(-2)^{0}$
$=-1+1$
$=0$
2c). $(3+2)^{0}$
$=(5)^{0}$
$=1$

## Writing Powers of Ten

Complete the missing values.

| Power | Repeated Multiplication | Standard Form | Words |
| :---: | :---: | :---: | :---: |
| $10^{3}$ | $10 \times 10 \times 10$ | 1000 | 1 thousand |
| $10^{5}$ | $10 \times 10 \times 10 \times 10 \times 10$ | 100000 | hundred thousand |
| $?$ | $10 \times 10 \times 10 \times 10 \times 10 \times 10$ | $?$ | 1 million |
| $10^{2}$ | $?$ | $?$ | $?$ |
| $?$ | 10 | 10 | ten |
| $10^{0}$ | $?$ | $?$ | $?$ |

## ANSWERS

| Power | Repeated Multiplication | Standard Form | Words |
| :---: | :---: | :---: | :---: |
| $10^{3}$ | $10 \times 10 \times 10$ | 1000 | 1 thousand |
| $10^{5}$ | $10 \times 10 \times 10 \times 10 \times 10$ | 100000 | hundred thousand |
| $10^{6}$ | $10 \times 10 \times 10 \times 10 \times 10 \times 10$ | 1000000 | 1 million |
| $10^{2}$ | $10 \times 10$ | 100 | 1 hundred |
| $10^{1}$ | 10 | 10 | ten |
| $10^{0}$ |  | 1 | one |

## Section 2.3 Order of Operations

Review the basics
Adding Integers
Subtracting Integers

$$
\begin{aligned}
& (+5)+(+2)=+7 \\
& (-6)+(-4)=-10 \\
& (-8)+(+2)=-6 \\
& (+9)+(-3)=+6
\end{aligned}
$$

$$
(+7)-(+3)=(+4)
$$

$$
(-6)-(-3)=(-6)+(+3)=-3
$$

$$
(-2)-(+9)=(-2)+(-9)=-11
$$

$$
(+3)-(-6)=(+3)+(+6)=+9
$$

When subtracting remember to
"Add the Opposite"

Multiplying Integers

$$
(+2)(+3)=+6
$$

$(-4)(-5)=+20$
$(+3)(-5)=-15$
$(-2)(+7)=-14$

Dividing Integers

$$
\begin{aligned}
& ++10) \div(+2)=+5 \\
& (-45) \div(-5)=+9 \\
& (-121) \div(+11)=-11 \\
& (+64) \div(-8)=-8
\end{aligned}
$$

When multiplying or dividing:
$\left.\begin{array}{r}+\times+=+ \\ -\times-=+\end{array}\right\} \quad \begin{aligned} & \text { same signs is } \\ & \text { positive }\end{aligned}$
$\left.\begin{array}{l}-\times+=- \\ +\times-=-\end{array}\right\} \begin{aligned} & \text { different signs is } \\ & \text { negative }\end{aligned}$

## Order of Operations

B - do operations inside brackets first
E - exponents
D multiply or divide, in order, from left to right, whichever comes first
M
$\left.\begin{array}{l}\text { A } \\ \text { S }\end{array}\right\}$ add or subtract, in order, from left to right, whichever comes first

Examples
A). $\quad 2^{3}+1$
$(2)(2)(2)+1$
$8+1$
9
C). $(3-1)^{3}$
$(2)^{3}$
8
B). $8-3^{2}$
$8-(3)(3)$
8-9
$8+-9$
$-1$
D). $\begin{aligned} & {\left[2 \times(-2)^{3}\right]^{2}} \\ & {[2 \times(-2)(-2)(-2)]^{2}}\end{aligned}$
$[2 \times(-8)]^{2}$
$[-16]^{2}$
$(-16)(-16)$
256
E).

$$
\begin{aligned}
& \left(7^{2}+5^{0}\right) \div(-5)^{1} \\
& {[(7)(7)+1] \div(-5)^{1}} \\
& {[49+1] \div(-5)^{1}} \\
& 50 \div-5 \\
& -10
\end{aligned}
$$

F). This student got the correct answer, but did not earn full marks. Find and explain the mistake the student made.

$$
\begin{aligned}
& -\left(24-3 \times 4^{2}\right)^{0} \div(-2)^{3} \\
& -\left(24-12^{2}\right)^{0} \div(-8) \\
& -(24-144)^{0} \div(-8) \\
& -(-120)^{0} \div(-8) \\
& -1 \div(-8) \\
& \quad \frac{1}{8}
\end{aligned}
$$

The mistake occurred at $4^{2} .4^{2}=16$ should have been done before $3 \times 4$. Or the student could have realized that the entire bracket has the exponent zero, so it's 1 .

$$
\begin{aligned}
& -\left(24-3 \times 4^{2}\right)^{0} \div(-2)^{3} \\
& -(1) \div(-2)^{3} \\
& -1 \div(-8) \\
& \quad \frac{1}{8}
\end{aligned}
$$

Section 2.4 Exponent Laws I
Product of Powers Investigation
1: Complete the table below.

| Product of Powers | Repeated Multiplication | Power Form |
| :---: | :---: | :---: |
| $10^{2} \times 10^{3}$ | $(10 \times 10) \times(10 \times 10 \times 10)$ | $10^{5}$ |
| $10^{3} \times 10^{4}$ |  |  |
| $5^{4} \times 5^{5}$ |  |  |
| $2^{3} \times 2^{1}$ |  |  |
| $3^{2} \times 3^{5}$ |  |  |
| $4^{3} \times 4^{2}$ |  |  |

2: Create 5 more examples of your own.

| Product of Powers | Repeated Multiplication | Power Form |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

3: State a rule for multiplying any two powers with the same base.

4: Can you use your rule to multiply $2^{3} \times 3^{2}$ ? Explain why or why not?

1: Complete the table below. ANSWERS

| Product of Powers | Repeated Multiplication | Power Form |
| :---: | :---: | :---: |
| $10^{2} \times 10^{3}$ | $(10 \times 10) \times(10 \times 10 \times 10)$ | $10^{5}$ |
| $10^{3} \times 10^{4}$ | $(10 \times 10 \times 10) \times(10 \times 10 \times 10 \times 10)$ | $10^{7}$ |
| $5^{4} \times 5^{5}$ | $(5 \times 5 \times 5 \times 5) \times(5 \times 5 \times 5 \times 5 \times 5)$ | $5^{9}$ |
| $2^{3} \times 2^{1}$ | $(2 \times 2 \times 2) \times(2)$ | $2^{4}$ |
| $3^{2} \times 3^{5}$ | $(4 \times 4 \times 4) \times(4 \times 4)$ | $3^{7}$ |
| $4^{3} \times 4^{2}$ |  | $4^{5}$ |

2: Create 5 more examples of your own.

| Product of Powers | Repeated Multiplication | Power Form |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

3: State a rule for multiplying any two powers with the same base.
$\hookrightarrow$ when multiplying powers with the same base, keep the base the same and add exponents. Base cannot be zero.

4: Can you use your rule to multiply $2^{3} \times 3^{2}$ ? Explain why or why not?
$\hookrightarrow$ No! The bases are NOT the same. Therefore, to evaluate this question you have to use BEDMAS.

1: Complete the table below.

| Quotient of Powers | Repeated Multiplication | Power Form |
| :---: | :---: | :---: |
| $10^{5} \div 10^{3}$ | $\frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10}$ | $10^{2}$ |
| $10^{8} \div 10^{5}$ |  |  |
| $5^{10} \div 5^{4}$ |  |  |
| $9^{8} \div 9^{3}$ |  |  |
| $7^{5} \div 7^{4}$ |  |  |
| $4^{7} \div 4^{4}$ |  |  |

2: Create 5 more examples of your own. Make sure you put the larger exponent first!

| Quotient of Powers | Repeated Multiplication | Power Form |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

3: $\quad$ State a rule for dividing two powers with the same base.

4: Can you use your rule to divide $5^{2} \div 2^{3}$ ? Explain why or why not?

1: Complete the table below.

| Quotient of Powers | Repeated Multiplication | Power Form |
| :---: | :---: | :---: |
| $10^{5} \div 10^{3}$ | $\frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10}$ | $10^{2}$ |
| $10^{8} \div 10^{5}$ | $\frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10 \times 10 \times 10}$ | $10^{3}$ |
| $5^{10} \div 5^{4}$ | $\frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5}$ |  |
| $9^{8} \div 9^{3}$ | $\frac{9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9}{9 \times 9 \times 9}$ | $5^{6}$ |
| $7^{5} \div 7^{4}$ | $\frac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7 \times 7}$ | $9^{5}$ |
| $4^{7} \div 4^{4}$ | $\frac{4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{4 \times 4 \times 4 \times 4}$ | $7^{1}$ |

2: Create 5 more examples of your own. Make sure you put the larger exponent first!

| Quotient of Powers | Repeated Multiplication | Power Form |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

3: $\quad$ State a rule for dividing two powers with the same base.
$\hookrightarrow$ when dividing powers with the same base, keep the base the same and subtract exponents. Base cannot be zero.

4: Can you use your rule to divide $5^{2} \div 2^{3}$ ? Explain why or why not?
$\hookrightarrow$ No! The bases are NOT the same. Therefore, to evaluate this question you have to use BEDMAS.

## Summary Notes

## Exponent Law for a Product of Powers

$$
\mathrm{a}^{\mathrm{m}} \times \mathrm{a}^{\mathrm{n}}=\mathrm{a}^{\mathrm{m}+\mathrm{n}}
$$

To multiply powers with the same base, (excluding a base of zero), keep the base and add the exponents.
where $\mathrm{a} \neq 0$ and m and n are whole numbers

1. Write as a single power, then evaluate.
a). $4^{3} \times 4^{4}$
b). $7^{5} \times 7^{-5}$
c). $(-3)^{2} \times(-3)^{4}$

$$
\begin{aligned}
4^{3+4} & =4^{7} \\
& =16384
\end{aligned}
$$

$$
\begin{aligned}
7^{5+(-5)} & =7^{0} \\
& =1
\end{aligned}
$$

$$
\begin{aligned}
(-3)^{2+4} & =(-3)^{6} \\
& =729
\end{aligned}
$$

2. Write as a single power.
a). $9^{5} \times 9$
b). $8^{-11} \times 8^{13}$
c). $3.8^{4} \times 3.8^{2}$
$9^{5+1}=9^{6}$
$8^{-11+13}=8^{2}$

$$
3.8^{4+2}=3.8^{6}
$$

d). $\left(\frac{1}{4}\right)^{12} \times\left(\frac{1}{4}\right)^{8}=\left(\frac{1}{4}\right)^{12+8}=\left(\frac{1}{4}\right)^{20} \quad$ e). $5^{2} \times 5 \times 5^{3}=5^{2+1+3}=5^{6}$

## Exponent Law for a Quotient of Powers

To divide powers with the same base, (excluding a base of zero), keep the base and subtract the exponents.

$$
a^{m} \div a^{n}=a^{m-n}
$$

where $\mathrm{a} \neq 0$ and m and n are whole numbers and $m \geq n$.
3. Write as a single power, then evaluate.
a). $2^{5} \div 2^{2}$
b). $\frac{(-6)^{8}}{(-6)^{6}}$
c). $\frac{3^{4}}{3^{4}}$

$$
2^{5-2}=2^{3}
$$

$$
(-6)^{8-6}=(-6)^{2}
$$

$$
=36
$$

$$
\begin{aligned}
3^{4-4} & =3^{0} \\
& =1
\end{aligned}
$$

4. Write as a single power.
a). $12^{6} \div 12$
b). $\frac{8^{3}}{8^{-2}}$
c). $(1.4)^{6} \div(1.4)^{2}$
$12^{6-1}=12^{5}$

$$
\begin{aligned}
8^{3-(-2)} & =8^{3+2} \\
& =8^{5}
\end{aligned}
$$

$$
(1.4)^{6-2}=1.4^{4}
$$

d). $\frac{\mathrm{x}^{7}}{\mathrm{x}^{5}}=x^{7-2}=x^{5}$
e). $\frac{5^{7}}{5^{3}}=5^{7-3}=5^{4}$

Note: "Evaluate" means to find the answer in "standard form"
Example: Evaluate $4^{3}=4 \times 4 \times 4=64$

$$
\text { Evaluate: } \quad \begin{array}{ll} 
& 2^{3} \times 2^{2} \\
& =2^{3+2} \\
& =2^{5} \\
& =32
\end{array}
$$

"Express as a single power" means leave your answer in "exponent form"

$$
\frac{5^{8}}{5^{2}}=5^{8-2}=5^{6}
$$

## Examples:

1. Express as a single power
a) $5^{2} \times 5^{4} \times 5$
$=5^{2+4+1}$
$=5^{7}$
b) $6^{-6} \times 6^{2}$
$=6^{-6+2}$
$=6-4$
c) $(-6)^{7} \div(-6)^{6}$
d) $10^{8} \div 10^{2}$
$=(-6)^{7-6}$
$=10^{8-2}$
$=(-6)$
$=10^{6}$
*** Often you will have problems where you will have to apply more than one exponent law.

$$
\text { e) } \begin{aligned}
& 8^{12} \div 8^{7} \times 8^{2} \\
& =8^{12-7} \times 8^{2} \\
& =8^{5} \times 8^{2} \\
& =8^{5+2} \\
& =8^{7}
\end{aligned}
$$

f) $\frac{2^{3} \times 2^{5}}{2^{2}}=\frac{2^{3+5}}{2^{2}}=\frac{2^{8}}{2^{2}}=2^{6}$

## Evaluate:

g) $\frac{(-4)^{10}}{(-4)^{3} \times(-4)^{3}}=\frac{(-4)^{10}}{(-4)^{3+3}}=\frac{(-4)^{10}}{(-4)^{6}}=(-4)^{10-6}=(-4)^{4}=256$

$$
\text { h) } \begin{aligned}
& 6^{2}+6^{3} \times 6^{2} \\
& =6^{2}+6^{3+2} \\
& =6^{2}+6^{5} \\
& =36+7776 \\
& =7812
\end{aligned}
$$

i) $(-10)^{4}\left[(-10)^{6} \div(-10)^{4}\right]-10^{7}$
$=(-10)^{4}\left[(-10)^{6-4}\right]-10^{7}$
$=(-10)^{4}(-10)^{2}-10^{7}$
$=(-10)^{4+2}-10^{7}$
$=(-10)^{6}-10^{7}$
$=1000000-10000000$
$=-9000000$

Section 2.5 Exponent Laws II

1: Complete the table below.

| Power of <br> a Power | Repeated <br> Multiplication | Product of Factors | Power <br> Form |
| :--- | :--- | :--- | :--- |
| $\left(2^{4}\right)^{3}$ | $2^{4} \times 2^{4} \times 2^{4}$ | $(2 \times 2 \times 2 \times 2) \times(2 \times 2 \times 2 \times 2) \times(2 \times 2 \times 2 \times 2)$ | $2^{12}$ |
| $\left(3^{2}\right)^{4}$ |  |  |  |
| $\left(4^{2}\right)^{3}$ |  |  |  |
| $\left(5^{3}\right)^{3}$ |  |  |  |
| $\left[(-4)^{3}\right]^{2}$ |  |  |  |
| $\left.[5)^{3}\right]^{5}$ |  |  |  |

2: $\quad$ State a rule for when you have two exponents (power of a power).

Section 2.5 Exponent Laws II ANSWERS Power of a Power Investigation 1
1: $\quad$ Complete the table below.
$\left.\begin{array}{|l|l|l|l|}\hline \begin{array}{l}\text { Power of } \\ \text { a Power }\end{array} & \begin{array}{l}\text { Repeated } \\ \text { Multiplication }\end{array} & \text { Product of Factors } & \begin{array}{l}\text { Power } \\ \text { Form }\end{array} \\ \hline\left(2^{4}\right)^{3} & 2^{4} \times 2^{4} \times 2^{4} & (2 \times 2 \times 2 \times 2) \times(2 \times 2 \times 2 \times 2) \times(2 \times 2 \times 2 \times 2) & 2^{12} \\ \hline\left(3^{2}\right)^{4} & 3^{2} \times 3^{2} \times 3^{2} \times 3^{2} & (3 \times 3) \times(3 \times 3) \times(3 \times 3) \times(3 \times 3) & 3^{8} \\ \hline\left(4^{2}\right)^{3} & 4^{2} \times 4^{2} \times 4^{2} & (4 \times 4) \times(4 \times 4) \times(4 \times 4) & 4^{6} \\ \hline\left(5^{3}\right)^{3} & 5^{3} \times 5^{3} \times 5^{3} & (5 \times 5 \times 5) \times(5 \times 5 \times 5) \times(5 \times 5 \times 5) & 5^{9} \\ \hline\left[(-4)^{3}\right]^{2} & \begin{array}{l}(-4)^{3} \times(-4)^{3} \\ {[(-4) \times(-4) \times(-4)] \times[(-4) \times(-4) \times(-4)]}\end{array} & (-4)^{6} \\ \hline\left[(-5)^{3}\right]^{5} & \begin{array}{l}(-5)^{3} \\ \times(-5)^{3} \times(-5)^{3} \\ \times(-5)^{3} \times(-5)^{3}\end{array} & \begin{array}{l}{[(-5) \times(-5) \times(-5)] \times[(-5) \times(-5) \times} \\ (-5) \times[(-5) \times(-5) \times(-5)] \times[(-5) \times\end{array} & (-5)^{15} \\ (-5) \times(-5) \times[(-5) \times(-5) \times(-5)]\end{array}\right]$

2: $\quad$ State a rule for when you have two exponents (power of a power).
$\hookrightarrow \quad$ when you have a power of a power, you keep the base the same and multiply the exponents. Base cannot be zero.

Section 2.5
Power of a Product Investigation 2
1: Complete the table below.

| Power | Repeated Multiplication | Product of Factors | Product <br> of Powers |
| :--- | :--- | :--- | :--- |
| $(2 \times 5)^{3}$ | $(2 \times 5) \times(2 \times 5) \times(2 \times 5)$ | $2 \times 2 \times 2 \times 5 \times 5 \times 5$ | $2^{3} \times 5^{3}$ |
| $(3 \times 4)^{2}$ |  |  |  |
| $(4 \times 2)^{5}$ |  |  |  |
| $(5 \times 3)^{4}$ |  |  |  |
| $(5 \times 6)^{2}$ |  |  |  |
| $[7 \times(-2)]^{3}$ |  |  |  |

2: State a rule for when you have a power of a product.

1: Complete the table below.

| Power | Repeated Multiplication | Product of Factors | Product of Powers |
| :---: | :---: | :---: | :---: |
| $(2 \times 5)^{3}$ | $(2 \times 5) \times(2 \times 5) \times(2 \times 5)$ | $2 \times 2 \times 2 \times 5 \times 5 \times 5$ | $2^{3} \times 5^{3}$ |
| $(3 \times 4)^{2}$ | $(3 \times 4) \times(3 \times 4)$ | $3 \times 3 \times 4 \times 4$ | $3^{2} \times 4^{2}$ |
| $(4 \times 2)^{5}$ | $\begin{gathered} (4 \times 2) \times(4 \times 2) \times(4 \times 2) \\ \times(4 \times 2) \times(4 \times 2) \end{gathered}$ | $\begin{gathered} 4 \times 4 \times 4 \times 4 \times \\ 2 \times 2 \times 2 \times 2 \end{gathered}$ | $4^{4} \times 2^{4}$ |
| $(5 \times 3)^{4}$ | $\begin{aligned} (5 \times 3) \times & (5 \times 3) \times(5 \times 3) \\ & \times(5 \times 3) \end{aligned}$ | $\begin{gathered} 5 \times 5 \times 5 \times 5 \times \\ 3 \times 3 \times 3 \times 3 \end{gathered}$ | $5^{4} \times 3^{4}$ |
| $(5 \times 6)^{2}$ | $(5 \times 6) \times(5 \times 6)$ | $5 \times 5 \times 6 \times 6$ | $5^{2} \times 6^{2}$ |
| $[7 \times(-2)]^{3}$ | $\begin{aligned} & {[7 \times(-2)] \times } {[7 \times(-2)] } \\ & \times[7 \times(-2)] \end{aligned}$ | $\begin{gathered} 7 \times 7 \times 7 \times \\ (-2) \times(-2) \times(-2) \end{gathered}$ | $7^{3} \times(-2)^{3}$ |

2: State a rule for when you have a power of a product.
$\hookrightarrow$ when you have a power of a product, the exponent outside the bracket is applied to each base inside the brackets. Base cannot be zero.

Section 2.5
Power of a Quotient Investigation 3

1: Complete the table below.


2: $\quad$ State a rule for when you have a power of a quotient.

1: $\quad$ Complete the table below.

| Power | Repeated Multiplication | Product of Factors | Product of <br> Quotients |
| :---: | :---: | :---: | :---: |
| $\left(\frac{5}{6}\right)^{3}$ | $\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6}$ | $\frac{5 \times 5 \times 5}{6 \times 6 \times 6}$ | $\frac{5^{3}}{6^{3}}$ |
| $\left(\frac{2}{3}\right)^{4}$ | $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ | $\frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3}$ | $\frac{2^{4}}{3^{4}}$ |
| $\left(\frac{1}{5}\right)^{5}$ | $\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5}$ | $\frac{1 \times 1 \times 1 \times 1 \times 1}{5 \times 5 \times 5 \times 5 \times 5}$ | $\frac{1^{5}}{5^{5}}$ |
| $\left(\frac{3}{10}\right)^{2}$ | $\frac{3}{10} \times \frac{3}{10}$ | $\frac{3 \times 3}{10 \times 10}$ | $\frac{3^{2}}{10^{2}}$ |
| $\left(\frac{-4}{7}\right)^{3}$ | $\left(\frac{-4}{7}\right) \times\left(\frac{-4}{7}\right) \times\left(\frac{-4}{7}\right)$ | $\frac{(-4) \times(-4) \times(-4)}{7 \times 7 \times 7}$ | $\frac{(-4)^{3}}{7^{3}}$ |
| $\left(\frac{-4}{-5}\right)^{6}$ | $\left(\frac{-4}{-5}\right) \times\left(\frac{-4}{-5}\right) \times\left(\frac{-4}{-5}\right) \times$ |  |  |
| $\left(\frac{-4}{-5}\right) \times\left(\frac{-4}{-5}\right) \times\left(\frac{-4}{-5}\right)$ | $\frac{(-4) \times(-4) \times(-4) \times(-4) \times(-4) \times(-4)}{(-5) \times(-5) \times(-5) \times(-5) \times(-5) \times(-5)}$ | $\frac{(-4)^{6}}{(-5)^{6}}$ |  |

2: State a rule for when you have a power of a quotient.
4 when you have a power of a quotient, the exponent outside the bracket is applied to the numerator and denominator inside the brackets. Base cannot be zero.

## Exponent Law for a Power of a Power

When you have a power to a power, the base stays the same and multiply the exponents.

$$
\left(a^{m}\right)^{n}=a^{m \times n}
$$

where $\mathrm{a} \neq 0$ and m and n are whole numbers

1. Write as a power.
a). $\left(3^{2}\right)^{4}$
b). $\left[(-7)^{3}\right]^{2}$
c). $-\left(2^{2}\right)^{4}$
$=3^{2 \times 4}$
$=(-7)^{3 \times 2}$
$=-\left(2^{2 \times 4}\right)$
$=3^{8}$
$=(-7)^{6}$
$=-2^{8}$
d). $\left(3^{0}\right)^{2}$
e). $\left(42^{3}\right)^{2} \times\left(42^{4}\right)^{4}$
$=3^{0 \times 2}$
$=42^{3 \times 2} \times 42^{4 \times 4}$
$=3^{0}$
$=42^{6} \times 42^{16}$

$$
=42^{6+16}
$$

$$
=42^{22}
$$

2. Simplify first, then evaluate.
a). $\left(2^{3}\right)^{2} \times\left(3^{2}\right)^{2}$
b). $\left(-3^{2}\right)^{3} \times\left(-3^{0}\right)^{9}$
$=2^{3 \times 2} \times 3^{2 \times 2}$
$=\left(-3^{2 \times 3}\right) \times\left(-3^{0 \times 9}\right)$
$=2^{6} \times 3^{4}$
$=\left(-3^{6}\right) \times\left(-3^{0}\right)$
$=64 \times 81$
$=\left(-3^{6}\right) \times-1$
$=5184$
$=-729 \times-1$
$=729$

## Exponent Law for a Power of a Product

When you have a power of a product, the exponent outside of the bracket is applied to the exponents on each of the factors on the inside of the brackets.
$(a b)^{m}=a^{m} b^{m}$
where $a \neq 0$ and $b \neq 0$ and $m$ is a whole number

1. Evaluate each question two ways. Use power of a product and BEDMAS.
a). $[(-7) \times 5]^{2}$
b). $-(3 \times 2)^{2}$

## Method 1:

## Method 1:

$$
\begin{array}{ll}
=(-7)^{2} \times 5^{2} & =-\left(3^{2} \times 2^{2}\right. \\
=49 \times 25 & =-(9 \times 4) \\
=1225 & =-36
\end{array}
$$

## Method 2:

$$
\begin{aligned}
& {[(-7) \times 5]^{2} } \\
= & {[-35]^{2} } \\
= & (-35) \times(-35) \\
= & 1225
\end{aligned}
$$

2. Evaluate, using any method of your choice.
a). $(3 \times 4)^{3}$
b). $\left[(-2)^{2} \times(-2)^{1}\right]^{3}$
$=\left[(-2)^{2+1}\right]^{3}$
$=1728$

$$
\begin{aligned}
& =\left[(-2)^{3}\right]^{3} \\
& =(-2)^{3 \times 3} \\
& =(-2)^{9}=-512
\end{aligned}
$$

## Exponent Law for a Power of a Quotient

When you have a power of a quotient, the exponent outside of the bracket is applied to the exponents on the numerator and denominator of the fraction inside of the brackets.

$$
\left(\frac{\mathrm{a}}{\mathrm{~b}}\right)^{\mathrm{m}}=\frac{\mathrm{a}^{\mathrm{m}}}{\mathrm{~b}^{\mathrm{m}}}
$$

where $a \neq 0$ and $b \neq 0$ and $m$ is a whole number

1. Evaluate each question two ways. Use power of a quotient and BEDMAS.
a). $[(-24) \div 6]^{4}$
b). $\left(\frac{52}{13}\right)^{3}$

Method 1:
$=(-24)^{4} \div 6^{4}$
$=331776 \div 1296$
$=256$

## Method 2:

$$
[(-24) \div 6]^{4}
$$

$=[-4]^{4}$
$=256$

Method 1:
$=\frac{(52)^{3}}{(13)^{3}}$
$=140608$
2197

$$
=64
$$

## Method 2:

$$
\left(\frac{52}{13}\right)^{3}
$$

$$
=(4)^{3}
$$

$$
=64
$$

